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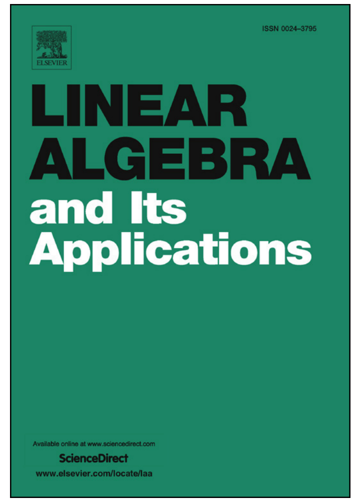

PII: S0024-3795(18)30290-8
DOI: https://doi.org/10.1016/j.laa.2018.06.006
Reference: LAA 14614

To appear in: *Linear Algebra and its Applications*

Received date: 20 December 2017
Accepted date: 4 June 2018

Convergence Analysis of an SVD-based Algorithm for the Best Rank-1 Tensor Approximation

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Abstract

This paper revisits the classical problem of finding the best rank-1 approximation to a generic tensor. The main focus is on providing a mathematical proof for the convergence of the iterates of an SVD-based algorithm. In contrast to the conventional approach by the so called alternating least squares (ALS) method that works to adjust one factor a time, the SVD-based algorithms improve two factors simultaneously. The ALS method is easy to implement, but suffers from slow convergence and easy stagnation at a local solution. It has been suggested recently that the SVD-algorithm might have a better limiting behavior leading to better approximations, yet a theory of convergence has been elusive in the literature. This note proposes a simple tactic to partially close that gap.

Keywords: best rank-1 tensor approximation, singular value decomposition
2010 MSC: 15A15, 15A09, 15A23

1. Introduction

A real-valued tensor of order-k can be represented by a k-way array

$$T = [\tau_{i_1,\ldots,i_k}] \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_k}$$

with elements $\tau_{i_1,\ldots,i_k}$ accessed via k indices. A tensor of the form

$$\bigotimes_{\ell=1}^{k} \mathbf{u}^{(\ell)} = \mathbf{u}^{(1)} \otimes \ldots \otimes \mathbf{u}^{(k)} := [u_{i_1}^{(1)} \ldots u_{i_k}^{(k)}],$$

where elements are the products of entries from vectors $\mathbf{u}^{(\ell)} \in \mathbb{R}^{I_\ell}$, $\ell = 1, \ldots, k$, is said to be of rank one. The problem of finding a best rank-1 approximation to T is to determine unit vectors $\mathbf{u}^{(\ell)} \in \mathbb{R}^{I_\ell}$, $\ell = 1, \ldots k$, and a scalar λ such that the functional

$$f(\lambda, \mathbf{u}^{(1)}, \ldots, \mathbf{u}^{(k)}) := \|T - \lambda \bigotimes_{\ell=1}^{k} \mathbf{u}^{(\ell)}\|_F^2 = \sum_{i_1, i_2, \ldots, i_k} (\tau_{i_1,\ldots,i_k} - \lambda u_{i_1}^{(1)} \ldots u_{i_k}^{(k)})^2 \quad (1)$$

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[1]This research is supported in part by the National Science Foundation under grant DMS-1316779.
[2]This research is supported in part by NUS research grant R-146-000-236-114.