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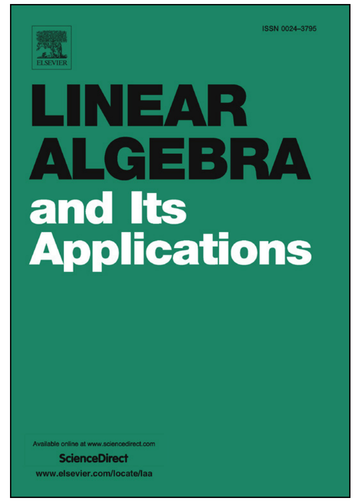

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The Algebraic Connectivity of a Graph and its Complement

B. Afshari[a], S. Akbari[b] *, M.J. Moghaddamzadeh[b], B. Mohar[c] †‡

[a]*School of Computer Science, Institute for Research in Fundamental Sciences,*

[b]*Department of Mathematical Sciences, Sharif University of Technology, Tehran, Iran,*

[c]*Department of Mathematics, Simon Fraser University, Burnaby, BC V5A 1S6*

Abstract

For a graph G, let $\lambda_2(G)$ denote its second smallest Laplacian eigenvalue. It was conjectured that $\lambda_2(G) + \lambda_2(\overline{G}) \geq 1$, where $\overline{G}$ is the complement of G. In this paper, it is shown that $\max\left\{\lambda_2(G), \lambda_2(\overline{G})\right\} \geq \frac{2}{5}$.

AMS Classification: 05C50
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1 Introduction

All graphs considered in this paper are simple (no loops and no multiple edges). If G is a graph and $v \in V(G)$, we denote by $N_G(v)$ the set of vertices adjacent to v. We denote the complementary graph of G by $\overline{G}$.

The adjacency matrix $A(G)$ of G is the matrix whose (u, v)-entry is equal to 1 if $uv \in E(G)$ and 0 otherwise. If $D(G)$ denotes the diagonal matrix of vertex degrees, then the Laplacian of the graph G is defined as $L(G) = D(G) - A(G)$. We denote the Laplacian eigenvalues of G by

$$0 = \lambda_1(G) \leq \lambda_2(G) \leq \cdots \leq \lambda_n(G).$$

The second smallest eigenvalue $\lambda_2(G)$ is also called the *algebraic connectivity* of G and is an important indicator related to various properties of the graph. It is well-known that

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‡On leave from IMFM & FMF, Department of Mathematics, University of Ljubljana.