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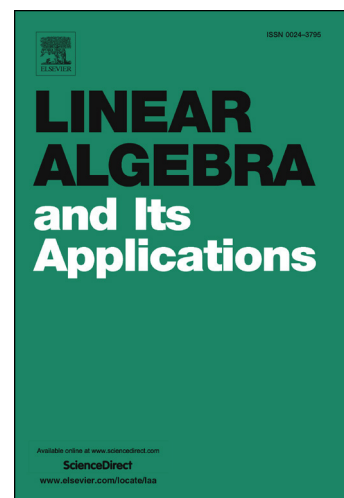
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Joint numerical ranges of operators in semi-Hilbertian spaces

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Abstract

In this paper we aim to investigate the concept of numerical range and maximal numerical range relative to a positive operator of a d -tuple of bounded linear operators on a Hilbert space. Some properties and applications of these sets are studied. Mainly, it is proved that they are convex for $d = 1$, this generalizes the well known Toeplitz-Hausdorff Theorem [24, 16] and Stampi's result [23]. Moreover, under additional hypotheses, we show that these sets are convex for $d \geq 2$.

Keywords. Positive operator, joint numerical range, maximal numerical range, convexity.

Mathematics Subject Classification Primary 46C05; Secondary 47A05.

1 Introduction and Preliminaries

Let \mathcal{H} be a non trivial complex Hilbert space with inner product $\langle \cdot | \cdot \rangle$ and associated norm $\| \cdot \|$. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of bounded linear operators on \mathcal{H} . For $T \in \mathcal{B}(\mathcal{H})$, the classical numerical range of T was introduced by Toeplitz in [24] as

$$W(T) := \{ \langle Tx | x \rangle; x \in \mathcal{H} \text{ with } \|x\| = 1 \}.$$

By the Toeplitz-Hausdorff Theorem, $W(T)$ is convex. This theorem has many proofs, a recent one is due to C.K. Li [18]. A short proof covering unbounded operators acting on a pre-Hilbert space was given by K. Gustafson [14]. Other basic properties of $W(T)$ can be found in [5]. There is a rich variety of generalizations of the notion of the numerical range. For example in [13], W. Givens has introduced the generalized numerical range of a matrix $M \in \mathbb{M}_n(\mathbb{C})$ by considering the generalized inner product induced by a positive definite hermitian matrix N . It is defined by

$$F_N(M) := \{ x^* N M x; x \in \mathbb{C}^n, x^* N x = 1 \}.$$

The concept of the classical numerical range was generalized to the joint numerical range by A.T. Dash [10] as follows.

Definition 1.1. ([10]) Let $\mathbf{T} = (T_1, \dots, T_d) \in \mathcal{B}(\mathcal{H})^d$ be a d -tuple of operators. The joint numerical range of \mathbf{T} is the subset of \mathbb{C}^d defined by

$$JtW(\mathbf{T}) = \{ (\langle T_1 x | x \rangle, \dots, \langle T_d x | x \rangle); x \in \mathcal{H}, \|x\| = 1 \}.$$

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