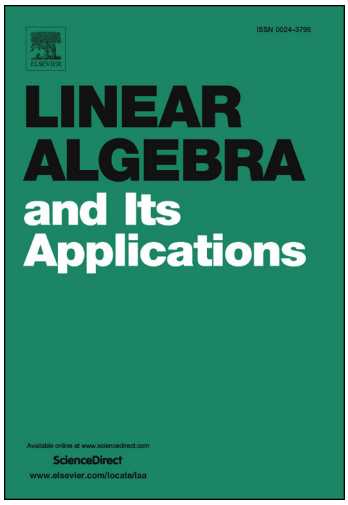

On the realizability of the critical points of a realizable list

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On the realizability of the critical points of a realizable list[☆]

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Abstract

The nonnegative inverse eigenvalue problem (NIEP) is to characterize the spectra of entrywise nonnegative matrices. A finite multiset of complex numbers is called realizable if it is the spectrum of an entrywise nonnegative matrix. Monov conjectured that the k^{th}-moments of the list of critical points of a realizable list are nonnegative. Johnson further conjectured that the list of critical points must be realizable. In this work, Johnson's conjecture, and consequently Monov's conjecture, is established for a variety of important cases including Ciarlet spectra, Suleĭmanova spectra, spectra realizable via companion matrices, and spectra realizable via similarity by a complex Hadamard matrix. Additionally we prove a result on differentiators and trace vectors, and use it to provide an alternative proof of a result due to Malamud and a generalization of a result due to Kushel and Tyaglov on circulant matrices. Implications for further research are discussed.

Keywords: Nonnegative inverse eigenvalue problem, Suleĭmanova spectrum, differentiator, trace vector, complex Hadamard matrix
2010 MSC: 15A29, 15A18, 15B48, 30C15

1. Introduction

The longstanding *nonnegative inverse eigenvalue problem* (NIEP) is to characterize the spectra of entrywise nonnegative matrices. More specifically, given a finite multi-set (herein *list*) $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ of complex numbers, the NIEP

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