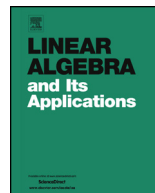




Contents lists available at ScienceDirect

# Linear Algebra and its Applications

[www.elsevier.com/locate/laa](http://www.elsevier.com/locate/laa)



## A congruence on the semiring of normal tropical matrices



Baomin Yu <sup>a,b,1</sup>, Xianzhong Zhao <sup>a,\*,2</sup>, Lingli Zeng <sup>a,3</sup>

<sup>a</sup> School of Mathematics, Northwest University, Xi'an, Shaanxi, 710127, PR China

<sup>b</sup> School of Mathematics and Physics, Weinan Normal University, Weinan, Shaanxi, 714099, PR China

### ARTICLE INFO

#### Article history:

Received 27 February 2018

Accepted 22 June 2018

Available online xxxxx

Submitted by R. Brualdi

#### MSC:

15A80

16Y60

#### Keywords:

Tropical algebra

Normal matrix

Congruence

Kleene star

Idempotent matrix

### ABSTRACT

We introduce and study a congruence  $\rho$  on the normal tropical matrix semiring  $M_n^N$ , which is relevant to the Kleene stars of normal tropical matrices. We prove that this congruence is a bisemilattice congruence and give an exact description of each  $\rho$ -class. In particular, we show that the  $\rho$ -class  $E\rho$  is an interval when  $E$  is a strongly regular normal tropical matrix. We also present a method using Floyd–Warshall algorithm to compute the greatest lower bound of  $E\rho$  in such case.

© 2018 Elsevier Inc. All rights reserved.

\* Corresponding author.

E-mail addresses: [bmyuu@hotmail.com](mailto:bmyuu@hotmail.com) (B.M. Yu), [zhaoxz@nwu.edu.cn](mailto:zhaoxz@nwu.edu.cn) (X.Z. Zhao), [zengll929@163.net](mailto:zengll929@163.net) (L.L. Zeng).

<sup>1</sup> The first author is supported by the Natural Science Basic Research Plan in Shaanxi Province of China (2017JM1044), the Science and Technique Foundation of Weinan city (2016KYJ-3-7) and National Natural Science Foundation of China (11501419).

<sup>2</sup> The second author is supported by National Natural Science Foundation of China (11571278).

<sup>3</sup> The third author is supported by National Natural Science Foundation of China (11701449).

### 1. Introduction and preliminaries

By a *semiring* we mean a nonempty set  $S$  with two binary operations  $\oplus$  (addition) and  $\otimes$  (multiplication) such that

- $(S, \oplus)$  is a commutative semigroup;
- $(S, \otimes)$  is a semigroup;
- $\otimes$  distributes over  $\oplus$ .

A semiring  $S$  is said to be *commutative* if  $a \otimes b = b \otimes a$  for all  $a, b \in S$ . An element  $e$  ( $\varepsilon$ , respectively) of a semiring  $S$  is called a *neutral element for multiplication* (a *neutral element for addition*, respectively) if  $a \otimes e = e \otimes a = a$  ( $a \oplus \varepsilon = a$ , respectively) for all  $a \in S$ . A semiring with a multiplicative neutral element  $e$  and an additive neutral element  $\varepsilon$  is called an *idempotent semiring* (also called *dioid*, see [1,11,19]) if  $a \oplus a = a$  for all  $a \in S$ . Notice that some authors (see [9,10] for instance) require that the additive neutral element  $\varepsilon$  is absorbing. However we do not require our semirings and idempotent semirings to satisfy this property. If we define the binary relation  $\leq$  on an idempotent semiring  $S$  by

$$a \leq b \iff a \oplus b = b, \tag{1.1}$$

then it is a partial order relation (see [1]). Also, it is compatible with  $\oplus$  and  $\otimes$ , that is, for any  $a, b, c \in S$ ,

$$a \leq b \Rightarrow a \oplus c \leq b \oplus c, a \otimes c \leq b \otimes c \text{ and } c \otimes a \leq c \otimes b. \tag{1.2}$$

The tropical semiring  $\overline{\mathbb{R}}$  is a commutative idempotent semiring (that is, a commutative dioid). It is the set  $\mathbb{R} \cup \{-\infty\}$  equipped with the operations of tropical addition  $a \oplus b = \max(a, b)$  and tropical multiplication  $a \otimes b = a + b$ , where 0 and  $-\infty$  are the multiplicative neutral element and the additive neutral element, respectively. Note that the partial order  $\leq$  on  $\overline{\mathbb{R}}$  defined by (1.1) is the usual order of real numbers. Tropical algebra (also called max-plus algebra) is the algebra developed over the tropical semiring. It provides an attractive way to describe and solve nonlinear problems appearing in areas such as combinatorial optimization [3], control theory [5], geometry [8,15], automata theory [21], etc. Many such problems are naturally expressed using systems of tropical linear equations and inequalities, and so tropical matrices have been intensively studied (see [1,3–5,12,15,16,21]).

As in conventional linear algebra, we can extend the operations  $\oplus$  and  $\otimes$  on the tropical semiring  $\overline{\mathbb{R}}$  to matrices. That is, if  $A = (a_{ij}), B = (b_{ij})$  and  $C = (c_{ij})$  are matrices over  $\overline{\mathbb{R}}$  of compatible sizes, then we write  $C = A \oplus B$  if  $c_{ij} = a_{ij} \oplus b_{ij}$  for all  $i, j$  and  $C = A \otimes B$  if  $c_{ij} = \bigoplus_k a_{ik} \otimes b_{kj}$  for all  $i, j$ . With respect to the tropical matrix addition  $\oplus$  and the tropical matrix multiplication  $\otimes$ , the set  $M_n$  of all  $n \times n$  tropical

Download English Version:

<https://daneshyari.com/en/article/8897729>

Download Persian Version:

<https://daneshyari.com/article/8897729>

[Daneshyari.com](https://daneshyari.com)