Accepted Manuscript

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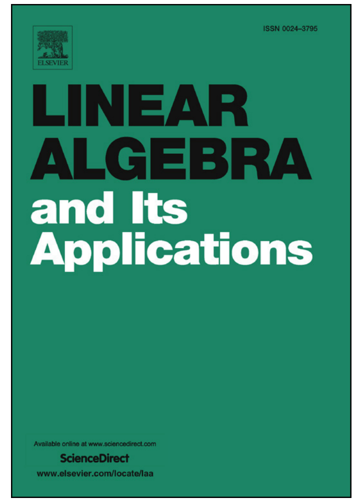

PII: S0024-3795(18)30254-4
DOI: https://doi.org/10.1016/j.laa.2018.05.015
Reference: LAA 14586

To appear in: *Linear Algebra and its Applications*

Received date: 25 December 2017
Accepted date: 16 May 2018

Zero-nonzero patterns of order $n \geq 4$ do not require $\mathbb{H}_n^*$

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Abstract

In [1], the set of refined inertias $\mathbb{H}_n$ for sign patterns is expanded to $\mathbb{H}_n^*$ for zero-nonzero patterns. Also, it is proved that there are no zero-nonzero patterns of even order $2m$ that require $\mathbb{H}_{2m}^*$. In this paper, it is shown that there are no zero-nonzero patterns of odd order $n \geq 5$ that require $\mathbb{H}_n^*$, which together with the previous result prove that there are no zero-nonzero patterns of order $n \geq 4$ that require $\mathbb{H}_n^*$.

AMS classification: 15B35, 15A18, 05C50, 05C20
Keywords: Eigenvalues; Refined inertia; Zero-nonzero pattern; Digraph.

1 Introduction

The set of refined inertias $\mathbb{H}_n$ is relevant to the study of dynamical systems, where the presence of nonzero pure imaginary eigenvalues can signal the onset of periodic solutions by Hopf bifurcation ([2]). Previous papers ([2]-[6] and references therein) have focused on sign patterns that require or allow $\mathbb{H}_n$. Recently, in [1], the concept $\mathbb{H}_n$ for sign patterns is expanded to $\mathbb{H}_n^*$ for zero-nonzero patterns.

An $n \times n$ *sign pattern* (*matrix*) is an $n \times n$ matrix whose entries come from the set $\{+, -, 0\}$. For a real matrix B, $\text{sgn}(B)$ is the sign pattern matrix obtained by replacing each positive (resp., negative) entry of B by $+$ (resp., $-$). For an $n \times n$ sign pattern matrix $\mathcal{A}$, the qualitative class of $\mathcal{A}$, denoted $Q(\mathcal{A})$, is defined as $Q(\mathcal{A}) = \{B \in M_n(\mathbb{R}) \mid \text{sgn}(B) = \mathcal{A}\}$. Two sign patterns are said to be *equivalent* if one can be obtained from the other by any combination of transposition, signature similarity, and permutation similarity.

An $n \times n$ *zero-nonzero pattern* is an $n \times n$ matrix $\mathcal{A}$ with entries from $\{*, 0\}$, where $*$ is nonzero. A real matrix $B = (b_{ij})$ is a *realization* of $\mathcal{A}$ if $b_{ij} \neq 0$ if and only if the corresponding entry of $\mathcal{A}$ is $*$. The *zero-nonzero pattern class* $Q(\mathcal{A})$ is the set of all realizations of $\mathcal{A}$. Two zero-nonzero patterns are *equivalent* if one can be obtained from the other by any combination of transposition and permutation similarity.

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