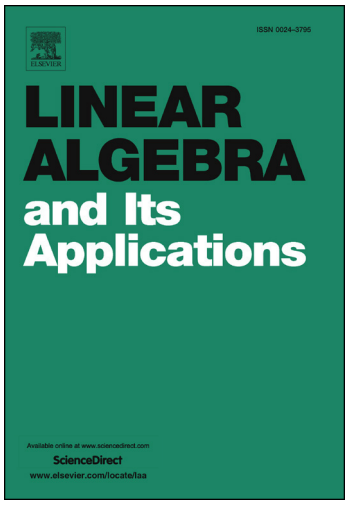

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On Laplacian energy, Laplacian-energy-like invariant and Kirchhoff index of graphs

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Abstract

Let G be a connected graph of order n and size m with Laplacian eigenvalues $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_n = 0$. The Kirchhoff index of G, denoted by Kf, is defined as: $Kf = n\sum_{i=1}^{n-1} \frac{1}{\mu_i}$. The Laplacian-energy-like invariant (LEL) and the Laplacian energy (LE) of the graph G, are defined as: $LEL = \sum_{i=1}^{n-1} \sqrt{\mu_i}$ and $LE = \sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|$, respectively. We obtain two relations on LEL with Kf, and LE with Kf. For two classes of graphs, we prove that $LEL > Kf$. Finally, we present an upper bound on the ratio LE/LEL and characterize the extremal graphs.

AMS classification: 05C50, 15A18

Keywords: Graph, Laplacian spectrum (of graph), Kirchhoff index, Laplacian–energy–like invariant, Laplacian energy

1 Introduction

Let $G = (V, E)$ be a simple graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$, where $|V(G)| = n$, $|E(G)| = m$. Let d_i be the degree of the vertex v_i for $i = 1, 2, \ldots, n$. The maximum degree is denoted by Δ. Let $A(G)$ be the (0, 1)-adjacency matrix of G and $D(G)$ be the diagonal matrix of vertex degrees. The Laplacian matrix of G is $L(G) = D(G) - A(G)$. The Laplacian matrix $L(G)$ has nonnegative eigenvalues $n \geq \mu_1 \geq \mu_2 \geq \cdots \geq \mu_n = 0$. Denote by $Spec(G) = (\mu_1, \mu_2, \ldots, \mu_n)$ the spectrum of $L(G)$, i. e., the Laplacian spectrum of G.

The *Laplacian energy* of the graph G is defined as [20]

$$LE = LE(G) = \sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|. \tag{1}$$