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Central limit theorem for linear spectral statistics of large dimensional quaternion sample covariance matrices ☆

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A B S T R A C T

Laguerre (Wishart) β-ensembles, where $\beta = 1, 2, 4$ (known as Dyson's threefold way) are important in Random Matrix Theory. These classical matrices models have been considered by many authors. In this paper, we investigate the central limit theorem for linear spectral statistics of large dimensional quaternion sample covariance matrices, which include the Wishart quaternion ensembles as a special case. Combining with existing results, a general central limit theorem, where $\beta = 1, 2, 4$ are parameters in the expressions of limit mean and variance, is also given.

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1. Introduction

Random matrix theory (RMT) is known to find applications in various branches of science. The study of random matrices emerge in 1920's with the publishing of Wishart's important work in 1928 [20]. Then in 1950's, Wigner suggest to replace the underlying physics of the problem by its statistical features, as we know little about the intricacies of complex atomic nuclei. This genius idea turns out to be surprisingly useful and finally make RMT popular. Generally speaking, classical random matrix theory mainly focuses on the so called Hermite (Gaussian) β-ensembles and Laguerre (Wishart) β-ensembles, where $\beta = 1, 2, 4$ (Dyson's threefold way [6]) corresponds to the invariance group of the ensemble (orthogonal, unitary or symplectic). Different with Hermite β-ensembles, which arise in physics, Wishart β-ensembles arise in statistics, and the three corresponding models for $\beta = 1, 2, 4$ could be named Wishart real, Wishart complex, and Wishart quaternion. The former two ensembles have been extensively researched while the last one was a less-studied random matrix model until recent years. The Wishart quaternion ensemble contains random matrices of the form $\mathbf{B} = \mathbf{X}^*\mathbf{X}$, where $\mathbf{X}$ is a rectangular quaternion matrix of size $p \times n$, whose entries are independent quaternion Gaussian variables, and $\mathbf{X}^*$ is the Hermitian quaternion conjugate of $\mathbf{X}$.

Quaternions was discovered by Hamilton in 1843 [10] as an extension of the system of complex numbers to four-dimensional space. And the use of quaternions in various practical applications is rapidly gaining in popularity at present, we refer to [14,19,18, 16,8,11] and references therein. Here are some basic definitions.

Definition 1.1 *(Quaternion).* Let a, b, c and d be four real numbers, $\mathbb{e}$, $\mathbb{i}$, $\mathbb{j}$ and $\mathbb{k}$ be ordered bases that satisfy the following multiplication table:

$$\mathbb{i}^2 = \mathbb{j}^2 = \mathbb{k}^2 = -\mathbb{e}, \quad \mathbb{i} = \mathbb{j}\mathbb{k} = -\mathbb{k}\mathbb{j}, \quad \mathbb{j} = \mathbb{i}\mathbb{k} = -\mathbb{k}\mathbb{i}, \quad \mathbb{k} = \mathbb{i}\mathbb{j} = -\mathbb{j}\mathbb{i}. \tag{1.1}$$

Then we call $q = a\mathbb{e} + b\mathbb{i} + c\mathbb{j} + d\mathbb{k}$ a **quaternion**. The real part is $\Re q = a\mathbb{e}$ and the image part is $\Im q = b\mathbb{i} + c\mathbb{j} + d\mathbb{k}$.

Here $\mathbb{e}$ plays a role as unit and usually assumed to has norm 1.

Definition 1.2 *(Quaternion conjugate).* The **quaternion conjugate** of q is defined as

$$q^Q = a\mathbb{e} - b\mathbb{i} - c\mathbb{j} - d\mathbb{k}.$$

Definition 1.3 *(Quaternion norm).* The **quaternion norm** of q is defined as

$$\|q\|_Q = \sqrt{a^2 + b^2 + c^2 + d^2}\,\mathrm{norm}\,(\mathbb{e}) = \sqrt{a^2 + b^2 + c^2 + d^2}.$$

A quaternion q is said to be a **unit quaternion** if $\|q\|_Q = 1$.