

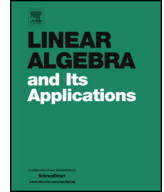


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A new aspect of Riordan arrays via Krylov matrices <sup>☆</sup>



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ABSTRACT

In this paper, we give a new angle to interpret Riordan arrays by showing that every Riordan array can be expressed as a Krylov matrix. We then use this idea to obtain some groups containing the Riordan group as a subgroup. Moreover, we study Lie algebras for the extended Riordan groups as Lie groups.

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1. Introduction

An  $n \times n$  matrix is said to be the *Krylov matrix* of a matrix  $A \in M_n(\mathbb{C})$  by a vector  $\mathbf{v} = (v_1, \dots, v_n)^T \in \mathbb{C}^n$  if its  $k$ th column is equal to  $A^{k-1}\mathbf{v}$ . We write  $K(A, \mathbf{v})$  for the matrix:

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$$K(A, \mathbf{v}) = \begin{bmatrix} \mathbf{v} & A\mathbf{v} & A^2\mathbf{v} & \cdots & A^{n-1}\mathbf{v} \end{bmatrix} \in M_n(\mathbb{C}). \tag{1}$$

Krylov matrices play a fundamental role in matrix computations. They are a key tool in understanding and developing numerical methods for solving eigenvalue problems of large sparse matrices and systems of linear equations.

If  $A$  is a triangular matrix then the Krylov matrix  $K(A, \mathbf{v})$  might have special underlying structures. For example, if  $A$  is a unit lower triangular matrix of the form:

$$A = \begin{bmatrix} 1 & & & & \\ a_{2,1} & 1 & & & O \\ \vdots & \ddots & \ddots & \ddots & \\ a_{n,1} & \cdots & a_{n,n-1} & 1 & \end{bmatrix},$$

it can be shown that for  $k \geq 1$ , the  $k$ th differences of  $k$ th row of the Krylov matrix  $K(A, \mathbf{v}) = [m_{ij}]$ :

$$(\Delta^{k-1}m_{k,1}, \Delta^{k-1}m_{k,2}, \dots, \Delta^{k-1}m_{k,n-k+1})$$

is the constant sequence  $(c_1, \dots, c_k)$  with  $c_1 = v_1$ ,  $c_k = v_1 a_{21} \cdots a_{k,k-1}$  for  $k \geq 2$  where  $\Delta^k m_{i,j} = \Delta^{k-1} m_{i,j+1} - \Delta^{k-1} m_{i,j}$  with  $\Delta^0 m_{i,j} = m_{i,j}$ . Further, the matrix  $K(A, \mathbf{v})$  can be factored into the product  $LU$  where  $L$  is a lower triangular matrix with diagonal entries  $c_1, c_2, \dots, c_n$  and  $U = P^T$  is the transpose of the  $n \times n$  Pascal matrix  $P$  whose  $(i, j)$ -entry is  $\binom{i-1}{j-1}$ . A similar observation has been used in [2]. It also follows that

$$\det K(A, \mathbf{v}) = v_1^n \cdot a_{2,1}^{n-1} \cdot a_{3,2}^{n-2} \cdots \cdots a_{n,n-1}^1. \tag{2}$$

In particular, if  $A$  is an infinite triangular matrix then the Krylov matrix  $K(A, \mathbf{v})$  is infinite and it is well-defined.

An infinite lower triangular matrix  $L = [\ell_{ij}]_{i,j \geq 0}$  is called a *Riordan array* or *Riordan matrix* generated by  $g$  and  $f$  if there exists a pair of formal power series  $(g, f)$ , with  $f(0) = 0$  over the complex field such that  $gf^j = \sum_{i \geq 0} \ell_{ij} z^i$  for  $j \geq 0$ . Usually we write  $L = \mathcal{R}(g, f)$  and it is written in terms of its column generating functions as follows:

$$\mathcal{R}(g, f) = \begin{bmatrix} g & gf & gf^2 & \cdots \end{bmatrix}. \tag{3}$$

The purpose of this paper is to use a new angle to interpret Riordan arrays as Krylov matrices based on a simple observation of (1) and (3). We then provide a new insight that should be useful for an extension of the group of Riordan arrays called the Riordan group. More specifically, in Section 2 we first show that every Riordan array can be expressed as a Krylov matrix. We use this idea in Section 3 to obtain extended Riordan groups. Finally, in Section 4 we study Lie algebras for the extended Riordan groups.

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