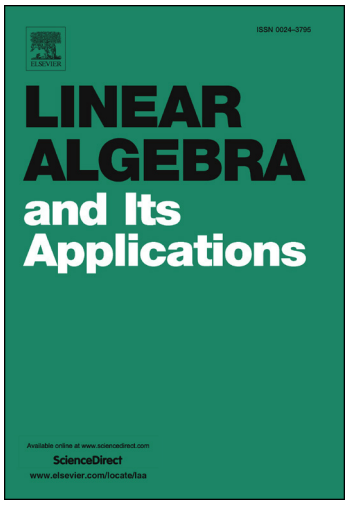

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PII: S0024-3795(18)30272-6
DOI: https://doi.org/10.1016/j.laa.2018.05.032
Reference: LAA 14604

To appear in: *Linear Algebra and its Applications*

Received date: 5 January 2018
Accepted date: 29 May 2018

Please cite this article in press as: M. Brugger et al., Additive colourful Carathéodory type results with an application to radii, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2018.05.032

ADDITIVE COLOURFUL CARATHÉODORY TYPE RESULTS WITH AN APPLICATION TO RADII

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Abstract. In this paper we study the behavior of the circumradius with respect to the Minkowski addition in generalized Minkowski spaces. To do so, we solve additive colourful Carathéodory type results, under certain equilibria conditions.

1. Introduction

Let us denote by $\mathcal{K}^n$ the set of all n-dimensional convex bodies, i.e., convex and compact sets.

The *circumradius* of a convex body K with respect to a second convex body C is the smallest rescalation λC containing a translation of K, and is denoted by $\mathrm{R}(K, C)$. The *inradius* of K w.r.t. C is the largest rescalation λC containing a translation of K, and is denoted by $\mathrm{r}(K, C)$. The *diameter* of K w.r.t. C is the maximum distance between two points of K measured w.r.t. $\|\cdot\|_{(C-C)/2}$, and is denoted by $\mathrm{D}(K, C)$. Finally, the *minimal width* of K w.r.t. C is the smallest distance between two parallel supporting hyperplanes to K measured w.r.t. $\|\cdot\|_{(C-C)/2}$, and is denoted by $\mathrm{w}(K, C)$. In the Euclidean space $(\mathbb{R}^n, \|\cdot\|_2)$ with unit ball $\mathbb{B}_2^n := \{x \in \mathbb{R}^n : \|x\|_2 \leq 1\}$, where $\|x\|_2 := \sqrt{x_1^2 + \cdots + x_n^2}$, and unit sphere $\mathbb{S}^{n-1} = \{x \in \mathbb{R}^n : \|x\|_2 = 1\}$, we write $\mathrm{R}(K) := \mathrm{R}(K, \mathbb{B}_2^n)$, and the same for $\mathrm{r}(K)$, $\mathrm{D}(K)$, and $\mathrm{w}(K)$.

The authors of [13, Theorems 1.1 and 1.2] initiated the study of the behavior of the successive radii (which are generalizations of the classical radii above) w.r.t. the Minkowski (or vectorial) addition in the Euclidean space with unit ball $\mathbb{B}_2^n$. In particular, the authors showed that for any two convex bodies K and L

$$\begin{aligned} &\mathrm{R}(K) + \mathrm{R}(L) \leq \sqrt{2}\mathrm{R}(K+L), \quad \mathrm{r}(K) + \mathrm{r}(L) \leq \mathrm{r}(K+L), \\ &\mathrm{D}(K) + \mathrm{D}(L) \leq \sqrt{2}\mathrm{D}(K+L), \quad \mathrm{w}(K) + \mathrm{w}(L) \leq \mathrm{w}(K+L). \end{aligned} \tag{1.1}$$

Other authors have studied the same questions for the different families of successive radii in the Euclidean space [10], for the mean successive radii [1], for the Firey (or p) sum [14], or for the Orlicz-Minkowski sum [8].

In this paper, however, our aim is to focus on the Minkowski addition of convex bodies, and to compute its behavior under the circumradius *measured with respect*

Date: May 30, 2018.

2010 *Mathematics Subject Classification.* Primary 52A20, Secondary 52A40.

Key words and phrases. Additive Helly theorem; Additive colourful Carathéodory theorem; Convex bodies; Outer and Inner radii; Vector sums; Minkowski addition.

The third author is partially supported by Fundación Séneca, through the Programa de Formación Postdoctoral de Personal Investigador, project reference 19769/PD/15, and the Programme in Support of Excellence Groups of the Región de Murcia, Spain, project reference 19901/GERM/15, and by MINECO project reference MTM2015-63699-P, Spain.