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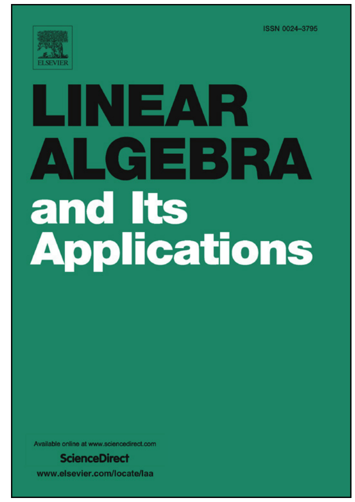

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SIGNED GRAPHS COSPECTRAL WITH THE PATH

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ABSTRACT. A signed graph Γ is said to be determined by its spectrum if every signed graph with the same spectrum as Γ is switching isomorphic with Γ. Here it is proved that the path P_n, interpreted as a signed graph, is determined by its spectrum if and only if $n \equiv 0, 1$, or $2 \pmod 4$, unless $n \in \{8, 13, 14, 17, 29\}$, or $n = 3$.
Keywords: signed graph; path; spectral characterization; cospectral graphs.
AMS subject classification 05C50, 05C22.

1. Introduction

Throughout this paper all graphs are simple, without loops or parallel edges. A *signed graph* $\Gamma = (G, \sigma)$ (with $G = (V, E)$) is a graph with the vertex set V and the edge set E together with a function $\sigma : E \to \{-1, +1\}$, called the *signature function*. So, every edge becomes either positive or negative. The adjacency matrix A of Γ is obtained from the adjacency matrix of the underlying graph G, by replacing 1 by -1 whenever the corresponding edge is negative. The spectrum of A is also called the spectrum of the signed graph Γ. For a vertex subset X of Γ, the operation that changes the sign of all outgoing edges of X, is called switching. In terms of the matrix A, switching multiplies the rows and columns of A corresponding to X by -1. The switching operation gives rise to an equivalence relation, and equivalent signed graphs have the same spectrum (see [9, Proposition 3.2]). If a signed graphs can be switched into an isomorphic copy of another signed graph, the two signed graphs are called *switching isomorphic*. Clearly switching isomorphic graphs are cospectral (that is, they have the same spectrum). A signed graph Γ is determined by spectrum whenever every graph cospectral with Γ is switching isomorphic with Γ. For unsigned graphs it is known that the path P_n is determined by the spectrum of the adjacency matrix, see [6, Proposition 1]. Among the signed graphs this is in general not true anymore. In this paper we determine precisely for which n this is still the case, see Theorems 4.5, 5.1, and Corollary 5.3.

We refer to [9] and [10] for more information about signed graphs. For the relevant background on graphs we refer to [3], [4], or [5]. The initial problem was, possibly, first introduced by Acharya in [1].

2. preliminaries

A *walk* of length k in a signed graph Γ is a sequence $v_1e_1v_2e_2 \dots v_ke_kv_{k+1}$ of vertices $v_1, v_2, \dots, v_{k+1}$ and edges $e_1, e_2, \dots, e_k$ such that $v_i \neq v_{i+1}$ and $e_i = \{v_i, v_{i+1}\}$ for each