

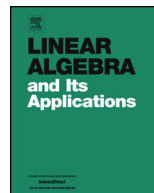


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Spectral characterizations of signed cycles



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ABSTRACT

A signed graph is a pair like (G, σ) , where G is the underlying graph and $\sigma : E(G) \rightarrow \{-1, +1\}$ is a sign function on the edges of G . In this paper we study the spectral determination problem for signed n -cycles (C_n, σ) with respect to the adjacency spectrum and the Laplacian spectrum. In particular, for the Laplacian spectrum, we prove that balanced odd cycles and unbalanced cycles, denoted, respectively, by C_{2n+1}^+ and C_n^- , are uniquely determined by their Laplacian spectra (i.e., they are DLS). On the other hand, we determine all Laplacian cospectral mates of the balanced even cycles C_{2n}^+ , so that we show that C_{2n}^+ is not DLS. The same problem is then considered for the adjacency spectrum, hence we prove that odd signed cycles, namely, C_{2n+1}^+ and C_{2n+1}^- , are uniquely determined by their (adjacency) spectrum (i.e., they are DS). Moreover, we find cospectral mates for the even signed cycles C_{2n}^+ and C_{2n}^- , and we show that, except the signed cycle C_4^- , even signed cycles are not DS and we provide almost all cospectral mates.

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1. Introduction and terminology

By a graph $G = (V, E)$, we mean a simple graph without loops or multiple edges, where $V = V(G)$ and $E = E(G)$ are the set of vertices and edges of G , respectively. Let $\sigma : E(G) \rightarrow \{-1, +1\}$ be a mapping defined on the edges of G . Then, the pair $\Gamma = (G, \sigma)$ is called a *signed graph*, where G is its underlying graph. We also say that G is an *unsigned* graph. If $v \in V$ is a vertex of Γ , then by $d_\Gamma(v)$ we mean the degree of v in G . A cycle in Γ is said to be *positive* if it contains an even number of negative edges, otherwise the cycle is called *negative*. A signed graph is said to be *balanced* if all its cycles are positive, otherwise it is *unbalanced*. Most of the usual graph theory glossary directly extends to signed graphs; for basic notation on signed graphs the reader is referred to [15]. Signed graphs were introduced by Harary [10] in connection with the study of the theory of social psychology. Among the fundamental works in this area, we can refer to Zaslavsky's papers [17] and [18]. Also, for a possibly complete bibliography on signed graphs see [14].

Now, let $\theta : V \rightarrow \{-1, +1\}$ be a sign function. The *switching* Γ by θ is a new signed graph $\Gamma^\theta = (G, \sigma^\theta)$, where $\sigma^\theta(e) = \theta(v_i)\sigma(e)\theta(v_j)$ for each edge $e = v_i v_j \in E(G)$. Two signed graphs Γ and Λ are *switching equivalent* and we write $\Gamma \sim \Lambda$ if there exists a switching function θ such that $\Lambda = \Gamma^\theta$. Obviously, \sim is an equivalence relation on signed graphs with the same underlying graph. It is worthy to notice that the signature switching does not affect the sign of the cycles, i.e., Γ and Γ^θ share the set of positive cycles. Additionally, the sign of non-cyclic edges (say, bridges) is irrelevant. Therefore, if G is a tree, then all signed graphs on G are switching equivalent. Also, for unicyclic graphs, there are exactly two different switching equivalent classes. In particular, C_n , the cycle of order n , has two switching equivalent classes; the positive (or, balanced) cycles and the negative (or, unbalanced) cycles which are denoted by C_n^+ and C_n^- , respectively.

Similarly to unsigned graphs, matrices can be used to study signed graphs. Let $\{v_1, \dots, v_n\}$ be the vertices of Γ . Then, the *adjacency matrix* of Γ , $A(\Gamma)$ or simply A , is defined as following: $[A]_{ij} = \sigma(v_i, v_j)a_{ij}$, where $a_{ij} = 1$ if v_i and v_j are adjacent and $a_{ij} = 0$ otherwise. The polynomial $\phi(\Gamma, \lambda) = \det(\lambda I - A) = \lambda^n + a_1^\sigma \lambda^{n-1} + \dots + a_{n-1}^\sigma \lambda + a_n^\sigma$ is called the *characteristic polynomial* of Γ and its roots are *eigenvalues* or *spectrum* of Γ , denoted by $\text{Spec}(\Gamma)$. Note that the characteristic polynomial of the null graph, the graph with no vertices and edges, is defined to be the constant polynomial 1.

Also, the matrix $L(\Gamma) = D(\Gamma) - A(\Gamma)$, or simply L , is called the *Laplacian matrix* of Γ , where $D(\Gamma)$ is the $n \times n$ diagonal matrix with $d(v_1), \dots, d(v_n)$ as the diagonal entries (and all other entries 0). The polynomial $\psi(\Gamma, \mu) = \det(\mu I - L)$ is called the *Laplacian polynomial* of Γ and its roots are *Laplacian eigenvalues* or *Laplacian spectrum* of Γ , denoted by $\text{Spec}_L(\Gamma)$. Observe that permuting the vertices in the underlying graph does not change the spectrum or the L -spectrum; a similar feature holds with signature switching: Γ and Γ^θ share the same adjacency or Laplacian spectrum. Therefore, if a signed graph can be switched into an isomorphic copy of another signed graph, then the two signed graphs are said to be *switching isomorphic* and they will get the same

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