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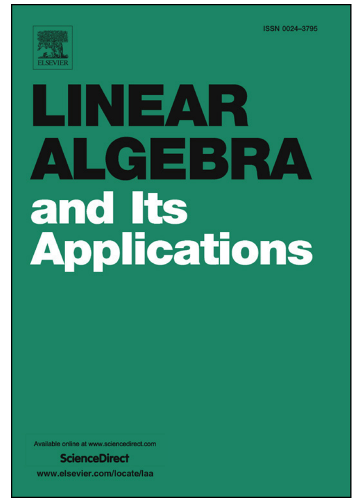

Block diagonal dominance of matrices revisited: Bounds for the norms of inverses and eigenvalue inclusion sets

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Block diagonal dominance of matrices revisited: bounds for the norms of inverses and eigenvalue inclusion sets

Dedicated to Richard S. Varga

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Abstract

We generalize the bounds on the inverses of diagonally dominant matrices obtained in [16] from scalar to block tridiagonal matrices. Our derivations are based on a generalization of the classical condition of block diagonal dominance of matrices given by Feingold and Varga in [11]. Based on this generalization we also derive a variant of the Gershgorin Circle Theorem for general block matrices which can provide tighter spectral inclusion regions than those obtained by Feingold and Varga.

Keywords: block matrices, block diagonal dominance, block tridiagonal matrices, decay bounds for the inverse, eigenvalue inclusion regions, Gershgorin Circle Theorem
2010 MSC: 15A45, 65F15

1. Introduction

Matrices that are characterized by off-diagonal decay, or more generally "localization" of their entries, appear in applications throughout the mathematical and computational sciences. The presence of such localization can lead to computational savings, since it allows to (closely) approximate a given matrix by using its significant entries only, and discarding the negligible ones according to a pre-established criterion. In this context it is then of great practical interest to know a priori how many and which of these entries can be discarded as insignificant. Many authors have therefore studied decay rates for different matrix classes and functions of matrices; see, e.g., [2, 4, 5, 6, 7, 10, 14, 18]. For an excellent survey of the current state-of-the-art we refer to [1].

An important example in this context is given by the (nonsymmetric) diagonally dominant matrices, and in particular the diagonally dominant tridiagonal matrices, which were studied, e.g., in [15, 16]. As shown in these works, the entries of the inverse decay with an exponential rate along a row or column, depending on whether the given matrix is row or column diagonally dominant;

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