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Bicyclic signed graphs with minimal and second minimal energy ☆

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ABSTRACT

Let $S = (G, \sigma)$ be a signed graph of order n and size m, where G is its underlying graph and σ is the sign function. A connected signed graph S is said to be bicyclic if $m = n + 1$. In this paper, we provide characterizations of bicyclic signed graphs with minimal and second minimal energy. We notice that there are gaps in the proofs of the main results in J. Zhang, H. Kan (2014) [32] and J. Zhang, B. Zhou (2005) [31] which deal with minimal energy of bicyclic graphs and we plug these gaps as a consequence of the results obtained for bicyclic signed graphs.

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1. Introduction

A signed graph is defined to be a pair $S = (G, \sigma)$, where $G = (V, E)$ is the underlying graph of S and $\sigma : E \to \{-1, 1\}$ is the signing function (or signature). Our signed graphs have simple underlying graphs. We denote the positive edge by a plain line and the negative edge by a dotted line. A graph can be considered to be a signed graph with each edge positive, thus signed graphs become a generalization of graphs. The sign of a signed cycle is defined as the product of signs of its edges. A signed cycle is said to be positive (respectively, negative) if its sign is positive (respectively, negative), that is, it contains an even (respectively, odd) number of negative edges. A signed graph is said to be balanced if each of its cycle is positive and unbalanced, otherwise. Throughout this paper, we denote by C_n^+ a positive cycle of order n, C_n^- a negative cycle of order n and C_n a cycle of order n which is either positive or negative. A connected signed graph of order n is said to be unicyclic or bicyclic according as the number of its edges is n or $n+1$. A basic figure is a graph whose components are cycles or edges or both. For undefined notations, we refer to [6,25].

The adjacency matrix of a signed graph S with vertex set $\{v_1, v_2, \cdots, v_n\}$ is the $n \times n$ matrix $A(S) = (a_{ij})$, where $a_{ij} = \sigma(v_i, v_j)$ if v_i is adjacent to v_j and zero, otherwise. As $A(S)$ is real symmetric, so has real eigenvalues. The characteristic polynomial $det(xI - A(S))$ of the adjacency matrix $A(S)$ of a signed graph S is called the characteristic polynomial of S and is denoted by $\phi(S, x)$. The eigenvalues of $A(S)$ are called the eigenvalues of S and the set of distinct eigenvalues together with their multiplicities is called its spectrum. If a signed graph of order n has $k \leq n$ distinct eigenvalues $x_1, x_2, \ldots, x_k$ with respective multiplicities $m_1, m_2, \ldots, m_k$, we write the spectrum as $\{x_1^{[m_1]}, x_2^{[m_2]}, \ldots, x_k^{[m_k]}\}$. Two signed graphs are said to be cospectral if they have the same spectrum and non-cospectral, otherwise. Acharya [1] proved that a signed graph S is balanced if and only if S and its underlying graph have same spectrum. For applications on signed graphs, see [12,20].

Gutman [11] defined the energy of a graph as the sum of the absolute values of eigenvalues of its adjacency matrix. Germina, Hameed and Zaslavsky [9] extended the concept of energy to signed graphs. The energy of a signed graph S with eigenvalues $x_1, x_2, \ldots, x_n$ is defined as $E(S) = \sum\limits_{j=1}^{n} |x_j|$. Some results on this topic can be found in [2,4,26–28]. A signed graph can be considered as a weighted graph with weights chosen from the set $\{-1, 1\}$. Gutman and Shao [13] defined the energy of a weighted graph with positive weights and studied some mathematical properties like energy change by deletion of edges. Given an $m \times n$ complex matrix M, its singular values are positive square roots of eigenvalues of MM^*, where M^* is the conjugate transpose of M. If $M = A(S)$, the adjacency matrix of S, the singular values become absolute values of eigenvalues of $A(S)$. If $x_1 \geq x_2 \geq \cdots \geq x_n$ are eigenvalues of S and $s_1 \geq s_2 \geq \cdots \geq s_n$ are singular values of S, where $s_j = |x_j|$, then $E(S) = \sum\limits_{j=1}^{n} s_j$. The energy of a matrix via singular values was first defined by Nikiforov [24]. Many energies of graphs were defined and among