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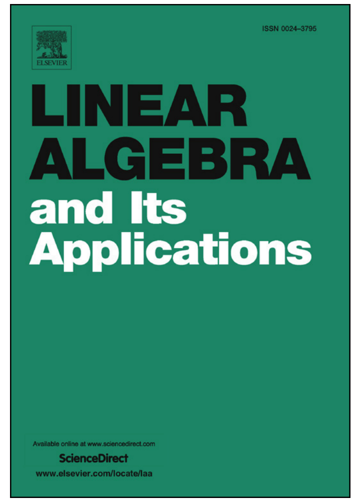


Improved bounds for the inverses of diagonally dominant tridiagonal matrices

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# IMPROVED BOUNDS FOR THE INVERSES OF DIAGONALLY DOMINANT TRIDIAGONAL MATRICES

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Abstract. We obtain new bounds for the entries of the inverse of a diagonally-dominant tridiagonal matrix which improve the best previous ones, due to H.-B. Li et al. We apply our bounds to the tridiagonal matrices arising in the second-order finite-difference discretization of certain boundary-value problems of parabolic type, establishing asymptotically optimal bounds.

## 1. Introduction

Tridiagonal matrices arise in connection with several scientific and technical problems. For example, the discretization by finite differences of second-order two-point boundary-value problems for ordinary differential equations requires the solution of linear systems of large size defined by tridiagonal matrices (see, e.g., [AMR95] or [LeV07]). In particular, conditioning and computation of inverses of nonsingular tridiagonal matrices have been the subject of many studies (see, e.g., [Hig02, §15.6]).

Explicit formulas for inverses of tridiagonal matrices are due to Gantmacher, Krein, Ikebe, Cao, Stewart, among others (see [Hig02, §15.7] for a brief historic account on these results). From such formulas one deduces an algorithm for computing all the entries of the inverse of a tridiagonal $n \times n$–matrix with $O(n)$ flops. Nevertheless, such a computation may break down, due to overflow and underflow (see [Hig02, §15.6]). This suggests that estimates for the entries to be computed may be relevant.

In this paper we shall be concerned with inverses of diagonally-dominant tridiagonal matrices. Such matrices has been intensively studied, and several estimates on its entries are available in the literature (see, e.g., [SJ96], [Nab98], [PP01], [LHLL10]). The best estimates, up to the authors knowledge, are due to [LHLL10].

Our main result establishes computable two-side bounds on the entries of the inverse of a real diagonally-dominant matrix which improve those of [LHLL10]. In fact, a comparison on two classes of tridiagonal matrices which arise in the discretization of certain unidimensional two-point boundary-value problems shows that there is an exponential gap between our bounds and those of [LHLL10]. We also determine the sign distribution and provide an efficient algorithm for computing the entries of the inverse of a given matrix.

Our approach relies on the analysis of the quotients $\alpha_i$ and $\beta_i$ of consecutive lower-right and upper-left principal minors of the matrix $A$ under consideration. We express the diagonal entries of the inverse matrix $A^{-1}$ in terms of these quantities, and the off-diagonal entries in terms of them and the diagonal entries of $A^{-1}$. We establish simple recursive formulas for the $\alpha_i$ and $\beta_i$ which may be evaluated in such a way as to furnish an efficient algorithm for computing the entries of $A^{-1}$. Further, the sign distribution of the entries of $A^{-1}$ is also obtained. As a byproduct of our approach, we obtain a simple characterization of nonsingular diagonally-dominant tridiagonal matrices.

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