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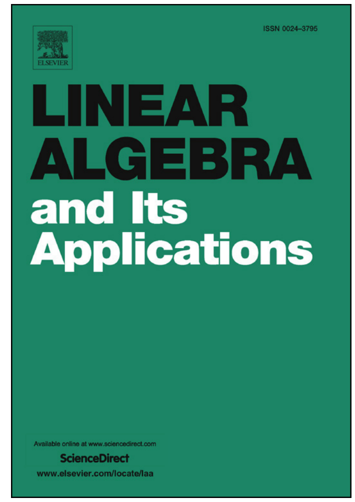

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The Inverse Eigenvalue Problem for Entanglement Witnesses

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Abstract

We consider the inverse eigenvalue problem for entanglement witnesses, which asks for a characterization of their possible spectra (or equivalently, of the possible spectra resulting from positive linear maps of matrices). We completely solve this problem in the two-qubit case and we derive a large family of new necessary conditions on the spectra in arbitrary dimensions. We also establish a natural duality relationship with the set of absolutely separable states, and we completely characterize witnesses (i.e., separating hyperplanes) of that set when one of the local dimensions is 2.

Keywords: quantum entanglement; eigenvalues; inverse problems; positive maps; absolute separability

MSC2010 Classification: 81P40; 65F18; 15A29

1 Introduction

In linear algebra and matrix theory, an inverse eigenvalue problem asks for a characterization of the possible spectra (i.e., the ordered tuples of eigenvalues) of a given set of matrices. Perhaps the most well-known inverse eigenvalue problem asks for the possible spectra of entrywise non-negative matrices [1]. This is called the non-negative inverse eigenvalue problem (NIEP), and it has been completely solved for matrices of size 4×4 and smaller, but remains unsolved for larger matrices.

Several variants of the NIEP have also been investigated, where instead a characterization is asked for of the possible spectra of *symmetric* non-negative matrices [2], stochastic matrices [3, 4], or doubly stochastic matrices [5, 6]. Similarly, the inverse eigenvalue problem has been considered for Toeplitz matrices [7] and tridiagonal matrices [8], among many others (see [9] and the references therein).

In this paper, we consider the inverse eigenvalue problem for entanglement witnesses, which are matrices of interest in quantum information theory that will be defined in the next section. Equivalently, we investigate what spectra can result from applying a positive matrix-valued map to just part of a positive semidefinite matrix. Such maps are of interest in operator theory (and again, we introduce the mathematical details in the next section).

The paper is organized as follows. In Section 2, we introduce the various mathematical tools that we will use throughout the paper. In Section 3, we completely solve the inverse eigenvalue problem for two-qubit entanglement witnesses (i.e., the lowest-dimension non-trivial case). In Section 4, we extend our investigation to qubit-qudit entanglement witnesses and obtain a large family of new necessary conditions on the spectra. In the process, we completely characterize the witnesses of the set of absolutely separable

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