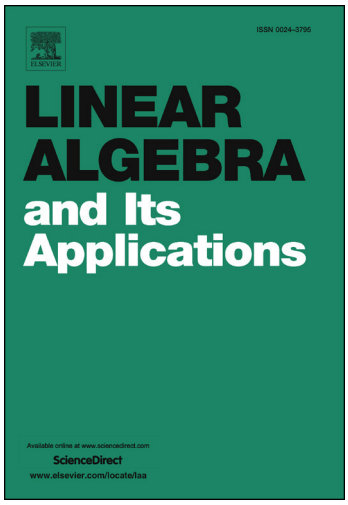

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On the α-index of graphs with pendent paths

Vladimir Nikiforov* and Oscar Rojo†

Abstract

Let G be a graph with adjacency matrix $A(G)$ and let $D(G)$ be the diagonal matrix of the degrees of G. For every real $\alpha \in [0,1]$, write $A_\alpha(G)$ for the matrix

$$A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G).$$

This paper presents some extremal results about the spectral radius $\rho_\alpha(G)$ of $A_\alpha(G)$ that generalize previous results about $\rho_0(G)$ and $\rho_{1/2}(G)$.

In particular, write $B_{p,q,r}$ be the graph obtained from a complete graph K_p by deleting an edge and attaching paths P_q and P_r to its ends. It is shown that if $\alpha \in [0,1)$ and G is a graph of order n and diameter at least k, then

$$\rho_\alpha(G) \le \rho_\alpha(B_{n-k+2,\lfloor k/2\rfloor,\lceil k/2\rceil}),$$

with equality holding if and only if $G = B_{n-k+2,\lfloor k/2\rfloor,\lceil k/2\rceil}$. This result generalizes results of Hansen and Stevanović [5], and Liu and Lu [7].

AMS classification: *05C50, 15A48*

Keywords: *convex combination of matrices; signless Laplacian; adjacency matrix; graph diameter; spectral radius.*

1 Introduction

Let G be a graph with adjacency matrix $A(G)$, and let $D(G)$ be the diagonal matrix of its vertex degrees. In [9] the matrix $A_\alpha(G)$ has been defined for any real $\alpha \in [0,1]$ as

$$A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G).$$

Write $Q(G)$ for the signless Laplacian $A(G) + D(G)$ of G and note that $A_0(G) = A(G)$ and $2A_{1/2}(G) = Q(G)$; thus, the family $A_\alpha(G)$ extends both $A(G)$ and $Q(G)$.

Write $\rho_a(G)$ for the spectral radius of $A_\alpha(G)$ and call $\rho_\alpha(G)$ the *α-index* of G. In the spirit of the general problem of Brualdi and Solheid [1], one can ask how large or how small can be the

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