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On the A_α-spectral radius of a graph

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ABSTRACT

Let G be a graph with adjacency matrix $A(G)$ and let $D(G)$ be the diagonal matrix of the degrees of G. For any real $\alpha \in [0, 1]$, Nikiforov [3] defined the matrix $A_\alpha(G)$ as

$$A_\alpha(G) = \alpha D(G) + (1 - \alpha)A(G).$$

The largest eigenvalue of $A_\alpha(G)$ is called the A_α-spectral radius of G. In this paper, we give three edge graft transformations on A_α-spectral radius. As applications, we determine the unique graph with maximum A_α-spectral radius among all connected graphs with diameter d, and determine the unique graph with minimum A_α-spectral radius among all connected graphs with given clique number. In addition, some bounds on the A_α-spectral radius are obtained.

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1. Introduction

All graphs considered here are simple and undirected. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The number $|V(G)|$ ($|E(G)|$) is the *order* (*size*) of G. The *neighborhood* $N(v)$ of a vertex v is $\{u \in V(G) : uv \in E(G)\}$ and the number $d_v = |N(v)|$ is the *degree* of v. If $S \subseteq V(G)$, then we use $G[S]$ to denote the subgraph of G induced by S. Let V_1 and V_2 be two sets of $V(G)$. We denote by $E[V_1, V_2]$ the set of edges of G with one end in V_1 and the other end in V_2. Let $G - v$ be a graph obtained from G by deleting v and all edges incident to v. We use $G \pm e$ to denote the graph obtained from G by adding/deleting the edge $e \notin E(G)/e \in E(G)$. Let G_1 and G_2 be two disjoint graphs. The graph $G_1 \cup G_2$ is the graph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. We denote by $G_1 \vee G_2$ the *join* of G_1 and G_2, which is the graph such that $V(G_1 \vee G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \vee G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1) \text{ and } v \in V(G_2)\}$. We denote by P_n, C_n and K_n the path, cycle and complete graph on n vertices, respectively.

Let G be a graph with vertex set $\{v_1, v_2, \dots, v_n\}$. The *adjacency matrix* of G is denoted by $A(G)$. The (i, j)-entry of $A(G)$ is 1 if $v_iv_j \in E(G)$, and otherwise 0. Let $D(G)$ be the diagonal matrix of the degrees of G. For any real $\alpha \in [0, 1]$, Nikiforov [3] defined the matrix $A_\alpha(G)$ as

$$A_\alpha(G) = \alpha D(G) + (1 - \alpha)A(G).$$

It is clear that $A_\alpha(G)$ is adjacency matrix if $\alpha = 0$, and $A_\alpha(G)$ is essentially equivalent to signless Laplacian matrix if $\alpha = 1/2$. We denote the eigenvalues of $A_\alpha(G)$ by $\lambda_1(A_\alpha(G)) \geq \lambda_2(A_\alpha(G)) \geq \cdots \geq \lambda_n(A_\alpha(G))$. The largest eigenvalue $\rho(G) := \lambda_1(A_\alpha(G))$ is called the A_α-spectral radius of G. More contents about A_α-matrix, one can see [3–5].

One of the central issues in spectral extremal graph theory is: *For a graph matrix, determine the maximization or minimization of spectral invariants over various families of graphs.* Many researchers have studied the analogous problems. Let P_k and P_l be two paths with $k \geq 1$, $l \geq 1$ and $d = k + l$. Let $K_{n-d}(k, l)$ be a graph obtained from a complete graph K_{n-d} by connecting all vertices of K_{n-d} to an end vertex of P_k and connecting all vertices to an end vertex of P_l, where $k + l = d$. In [2], van Dam proved that $K_{n-d}(\lfloor \frac{d}{2} \rfloor, \lceil \frac{d}{2} \rceil)$ attains the maximum A-spectral radius among all graphs with diameter d. Wang and Huang [9] presented that $K_{n-d}(\lfloor \frac{d}{2} \rfloor, \lceil \frac{d}{2} \rceil)$ also attains the maximum $A_{\frac{1}{2}}$-spectral radius among all graphs with diameter d. We generalize their results to $0 \leq \alpha < 1$.

Theorem 1.1. *Let $0 \leq \alpha < 1$. If G is a connected graph with diameter $d \geq 2$, then*

$$\rho(G) \leq \rho(K_{n-d}(\lfloor \frac{d}{2} \rfloor, \lceil \frac{d}{2} \rceil)),$$

where equality holds if and only if $G \cong K_{n-d}(\lfloor \frac{d}{2} \rfloor, \lceil \frac{d}{2} \rceil)$.

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