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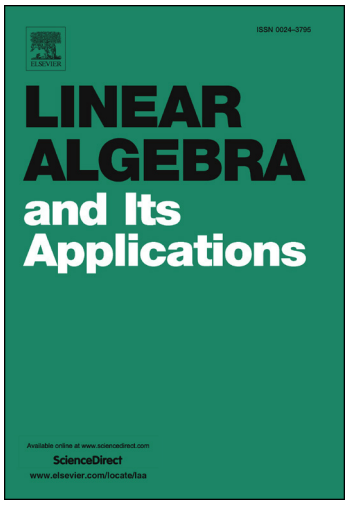

On the kernel of integral circulant graphs

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On the kernel of integral circulant graphs

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Abstract

Given a positive integer n, every integral circulant graph on n vertices is isomorphic to some graph $\mathrm{ICG}(n, \mathcal{D})$ having vertex set $\mathbb{Z}/n\mathbb{Z}$ and edge set $\{(a, b) : a, b \in \mathbb{Z}/n\mathbb{Z}, \gcd(a-b, n) \in \mathcal{D}\}$ for a uniquely determined set $\mathcal{D}$ of positive divisors of n. By virtue of its adjacency matrix, one defines the spectrum of a graph G and, naturally, can ask to which degree the eigenvalues of G determine the graph itself. With respect to integral circulant graphs little is known about this question, which is related to a conjecture of So.

In this note we examine the role of the eigenvalue 0 and clarify the interrelation between the dimension of the kernel of $\mathrm{ICG}(n, \mathcal{D})$ and the graph itself for all prime powers $n = p^k$ and for all positive integers n in case $\mathcal{D}$ is a multiplicative divisor set.

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Keywords: Cayley graph, integral graph, circulant graph, eigenvalue, graph spectrum, kernel, multiplicative divisor set, Conjecture of So.

1 Introduction

"Can one hear the shape of a drum?" asked Kac [7] in 1966, and this question has become a synonym for the problem to decide whether a given Riemannian manifold is determined by its spectrum. In [6] Fisher formulated the discrete analogue of Kac's question and thus transferred it to the examination of spectra of linear graphs by use of their adjacency matrices. Since the mid-twentieth century one has tried to find out which graphs are determined by their spectrum, and some answers were given for different types of graphs (cf. [5] for a survey, and [2] for graph spectra in general).