

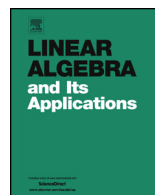


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Linear Algebra and its Applications

www.elsevier.com/locate/laa



A generalization of the Kreiss Matrix Theorem



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ARTICLE INFO

Article history:

Received 20 August 2017

Accepted 7 March 2018

Available online 16 March 2018

Submitted by P. Semrl

MSC:

47A10

47A30

47A60

30E10

Keywords:

Kreiss Matrix Theorem

Resolvent

Markov function

Faber polynomial

ABSTRACT

Let Ω be a compact subset of the complex plane such that its complement is simply connected in the extended complex plane. Suppose A is a linear bounded operator in a Hilbert space, with spectrum $\sigma(A) \subset \Omega$. If Ω is symmetric with respect to the real line and f is a Markov function, we show that

$$\|f(A)\| \leq eC \mathcal{K}(\Omega) \|f\|_{\Omega},$$

where $\mathcal{K}(\Omega)$ is the Kreiss constant with respect to Ω and C is a constant. We also present other extensions of the Kreiss Matrix Theorem to arbitrary holomorphic functions.

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1. Introduction and preliminary results

Let A be an $N \times N$ complex matrix with spectrum $\sigma(A)$, and let $\|\cdot\|$ denotes the operator norm which is defined with respect to the usual Euclidean norm on vectors. The original statemet of the Kreiss Matrix Theorem (KMT), a well known fact in applied

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matrix analysis, characterizes power bounded and exponentially bounded matrices. If $\sigma(A)$ is in the closed unit disc $\overline{\mathbb{D}}$, this celebrated theorem is known by the following inequality

$$\mathcal{K}(A) \leq \sup_{n \geq 0} \|A^n\| \leq e N \mathcal{K}(A), \quad (1)$$

where

$$\mathcal{K}(A) := \sup_{|z| > 1} (|z| - 1) \left\| (zI - A)^{-1} \right\|$$

is the Kreiss constant with respect to the closed unit disc. If $\sigma(A)$ is in the left half-plane, then the analogous result for matrix exponentials is the following inequality

$$\mathcal{K}(A) \leq \sup_{t \geq 0} \|e^{tA}\| \leq e N \mathcal{K}(A), \quad (2)$$

where here $\mathcal{K}(A)$ is the Kreiss constant with respect to the closed left half-plane and defined by

$$\mathcal{K}(A) := \sup_{\operatorname{Re}(z) > 0} (\operatorname{Re}(z)) \left\| (zI - A)^{-1} \right\|.$$

The constant eN , in the upper bounds in the inequalities (1) and (2), was obtained after several developments of the original statement in 1962, see [5]. The original proof shows the upper bound $\mathcal{K}(A)^{N^N}$. Then improvements were made by Morton in 1964, Strang in 1966 and Miller in 1967, which decreased the upper bound to $6^N(N+4)^{5N}\mathcal{K}(A)$, $N^N\mathcal{K}(A)$ and $e^{9N^2}\mathcal{K}(A)$, respectively. From a paper by Laptev, Tadmor showed in 1981 that this upper bound is linear in N and reduced it to $32eN\mathcal{K}(A)$. Leveque and Trefethen minimized this upper bound in 1984 to $2eN\mathcal{K}(A)$ and they conjectured that the optimal bound is $eN\mathcal{K}(A)$. In 1985, Smith reduced this upper bound to $(1 + \frac{2}{\pi})eN\mathcal{K}(A)$ and finally Spijker proved the conjecture in 1991. The proof of the optimal upper bound is based on a lemma, which provides an upper bound for the arc length of the image of the unit circle in the complex plane by a rational function. For details, see [6,8,10] and their references therein.

Many applications in science require the evaluation of the norm of $f(A)$ for an arbitrary holomorphic function f on a neighborhood of the spectrum of A . An interesting connection between $f(A)$ and the resolvent $(zI - A)^{-1}$ is given by the Cauchy integral formula. So, it is important to study the norm of the resolvent to understand $f(A)$. Recently, it has been shown that the norm of the resolvent determines the norm behavior of $f(A)$ if f is a Möbius transformation. However, it is not sufficient for general holomorphic functions, see [7] for details. The idea of the generalization of KMT is to get an appropriate upper bound of $\|f(A)\|$ if we have an appropriate upper bound of the resolvent norm. In [9], KMT was extended to Faber polynomials on a general complex

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