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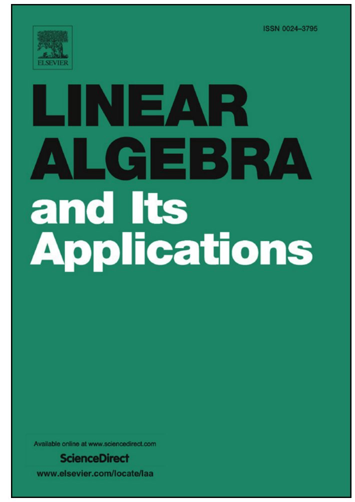

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Pauli group: Classification and Joint higher rank numerical range

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Abstract. In [2015, Joint higher rank numerical range of Pauli group, Linear and Multilinear Algebra, 63(3), 439-454], authors showed that joint higher rank numerical range of triple of Pauli group elements is often convex, and characterized it exactly in the case that it is convex. In the present paper, those results are derived in a different manner. Also, $N-$ qubit Pauli group using simultaneously unitary similarity are classified, which may generally be used to charactrize joint higher rank numerical range of any Hermitian $m-$tuple in the N-qubit Pauli group. Besides, some more results about joint numerical range are here derived.

Key words: Quantum error correction, Pauli channels, Joint higher rank numerical range, Joint numerical range, Hermitian matrices, Pauli group.

AMS Subject Classification 15A60, 47A12, 47N50, 81P68.

1 Introduction

Quantum computation is a fast-growing and a multi-disciplinary research field. If large-scale quantum computers can be manufactured, they will be able to solve certain problems much faster than any current classical ones. For example, Shor's algorithm [20] can be used to break public-key cryptography schemes such as the widely used RSA scheme. One of the biggest hurdles faced by quantum computing researchers is called "decoherence" (i.e. the tendency of quantum systems to be disturbed). There are several proposals to resolve this. Quantum error correction codes (QECC) along with the celebrated Knill-Laflamme (KL) conditions [10] totally form one of the most promising candidates to suppress environmental noise, which leads to decoherence [5]. KL conditions assert that given a quantum channel $\Phi : M_n \to M_n$ with error operators $\{E_i\}_{1\leq i,j\leq r}$, subspace $\mathcal{V} \subset \mathbb{C}^n$ is a QECC of Φ if and only if there exist scalars λ_{ij} and orthogonal projection operator $P \in M_n$with the range space $\mathcal{V}$ such that $PE_i^*E_jP = \lambda_{ij}P$.

KL conditions prompted the authors of the refrences [2, 3, 4, 6] to introduce the notion of (joint) higher rank numerical range of an ($m-$tuple of) operator(s). For $1 \leq k < n$ joint rank$-k$ numerical range of $A_1, \ldots, A_m \in M_n$ is defined by

$$\Lambda_k\left(A_1, \ldots, A_m\right) = \left\{(\lambda_1, \ldots, \lambda_m) : \exists U \in M_{k,n}, UU^* = I_k, \forall_j, UA_jU^* = \lambda_j I_k\right\}.$$

In fact, there is a QECC of dimension k for quantum channel Φ described above if and only if $\Lambda_k\left(A_1, \ldots, A_m\right)$ is non-empty for $\left(A_1, \ldots, A_m\right) = \left(E_1^*E_1, E_1^*E_2, \ldots, E_r^*E_r\right)$.

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