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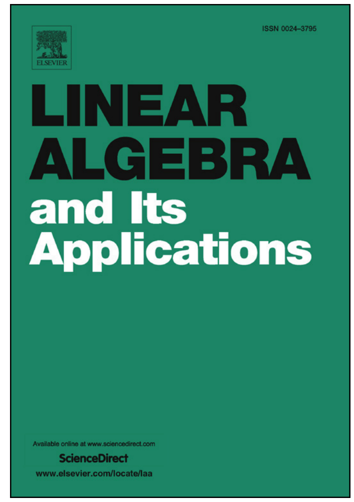

Some characterizations of minimal compact normal operators

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Some characterizations of minimal compact normal operators *

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Abstract In this note, we consider properties of a compact normal operator A such that

$$\|A\| \leqslant \|A+D\| \quad \text{for all } D \in \mathcal{D}(K(\mathcal{H})),$$

or equivalently,

$$\|A\| = \text{dist}(A, \mathcal{D}(K(\mathcal{H}))),$$

where $\|\cdot\|$ denotes the usual operator norm and $\mathcal{D}(K(\mathcal{H}))$ is the subalgebra of diagonal compact operators in a fixed orthonormal basis of the Hilbert space $\mathcal{H}$.

Keywords: Minimal compact operator, Quotient operator norm, Diagonal operators
Mathematics Subject Classification: 47A58, 47B15, 47A55

1 Introduction

Let $\mathcal{H}$ be a complex separable Hilbert space and $\mathcal{B}(\mathcal{H})$ be the set of all bounded linear operators on $\mathcal{H}$. For an operator $A \in \mathcal{B}(\mathcal{H})$, the adjoint of A is denoted by A^*. We write $A \geq 0$ if A is a positive operator, meaning $\langle Ax, x\rangle \geq 0$, where we denote by $\langle , \rangle$ the inner product of $\mathcal{H}$. Let $K(H)$ be the two sided closed ideal of compact operators on $\mathcal{H}$ and $T(H)$ be the set of trace class operators. Also, $|A| := (A^*A)^{\frac{1}{2}}$ denotes the absolute value of $A \in \mathcal{B}(H)$ and $\|X\|_1 = tr(|X|) = tr((X^*X)^{\frac{1}{2}})$ the trace norm of $X \in T(H)$ (see [10]). Let $\{e_k\}_{k=1}^{\infty}$ be a fixed orthonormal basis of $\mathcal{H}$ and $\mathcal{D}(\mathcal{B}(\mathcal{H}))$ be the subalgebra of diagonal operators in the orthonormal basis $\{e_k\}_{k=1}^{\infty}$. That is

$$\mathcal{D}(\mathcal{B}(\mathcal{H})) := \{T \in A : \langle Te_i, e_j\rangle = 0, \text{ for all } i \neq j\}.$$

Similarly, $\mathcal{D}(K(\mathcal{H})^h)$ represents the subalgebra of diagonal compact self-adjoint operators in the above orthonormal basis.

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