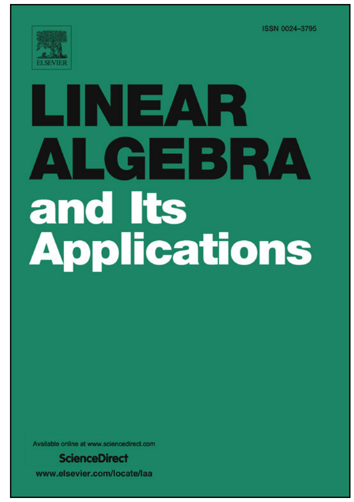

Lower bounds of graph energy in terms of matching number

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Lower bounds of graph energy in terms of matching number *

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Abstract: The energy $\mathcal{E}(G)$ of a graph G is the sum of the absolute values of all eigenvalues of G. We are interested in the relation between the energy of a graph G and the matching number $\mu(G)$ of G. It is proved that $\mathcal{E}(G) \geq 2\mu(G)$ for every graph G, and $\mathcal{E}(G) \geq 2\mu(G) + \frac{\sqrt{5}}{5}c_1(G)$ if the cycles of G (if any) are pairwise vertex-disjoint, where $c_1(G)$ denotes the number of odd cycles in G. Besides, we prove that $\mathcal{E}(G) \geq r(G) + \frac{1}{2}$ if G has at least one odd cycle and it is not of full rank, where $r(G)$ is the rank of G.

AMS classification: 05C20, 05C50, 05C75.

Keywords: graph energy; matching number; rank.

1 Introduction

Throughout, G denotes a simple graph, i.e., a graph with no loop and no multiple edge. Let $V(G)$ and $E(G)$ denote the vertex set and edge set of G, respectively, and let $A(G)$ denote the adjacency matrix of G. The rank of $A(G)$ is also called the rank of G and is denoted by $r(G)$. If $x, y \in V(G)$ are adjacent we denote by xy the edge joining x and y. A graph H is called a subgraph of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. Further, H is called an induced subgraph of G if two vertices of $V(H)$ are adjacent in H if and only if they are adjacent in G. The neighbors of a vertex $x \in V(G)$ in an induced subgraph H of G is written as $N_H(x)$ and $N_G(x)$ is simplified as $N(x)$. The number of vertices in $N(x)$ is called the degree of x in G, which is written as $d(x)$. If $d(x) = 1$, we call x a pendant vertex of G, and the unique neighbor of a pendant vertex is called a quasi-pendant vertex. For a subset U of $V(G)$, denote by $G - U$ the graph obtained from G by deleting the vertices of U together with all edges incident to them. If H is an induced subgraph of G, we will use $G - H$ to denote the induced subgraph $G - V(H)$. The subgraph $G - H$ of G is also called the complement of H in G. Moreover, when no edge of G joins H and its complement $G - H$, we write $G = H \oplus (G - H)$. For an induced subgraph H of G and a vertex x outside H, the induced subgraph of G with vertex set $V(H) \cup \{x\}$ is simply written as $H + x$. A matching M in G is a set of pairwise non-adjacent edges, that is, no

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