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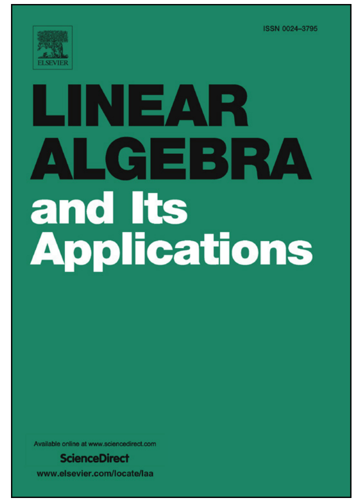

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ON SKEW LAURENT POLYNOMIAL RINGS OVER LOCALLY NILPOTENT RINGS

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ABSTRACT. Let R be a locally nilpotent ring. Let σ be an endomorphism of R and let δ be a σ-derivation of R. Then the skew Laurent polynomial ring $R[X, X^{-1}; \sigma, \delta]$ does not contain a non-zero idempotent. This answers a question posed by Greenfeld, Smoktunowicz and Ziembowski.

1. Introduction

In our paper [3] we suggested that the Linear Algebraic methods can be of a great use for some important problems in Ring Theory including the most famous open problem known as the Koethe Conjecture. The purpose of this paper is to show how a triangularization technique developed by Mesyan [5] can be applied to solve a problem posed by Greenfeld, Smoktunowicz and Ziembowski [4].

Let σ be an endomorphism of a ring R and let δ be a σ-derivation, that is $\delta : R \to R$ is an additive map such that

$$\delta(ab) = \sigma(a)\delta(b) + \delta(a)b, \quad \text{for all } a, b \in R.$$

Now consider the set of finite sums $\sum_{i=m}^{n} a_i X^i$ where $m \leq n$ are integers, addition is defined componentwise, and multiplication is given by the multiplication in R, the distributive law and the condition

$$Xa = \sigma(a)X + \delta(a), \quad \text{for all } a \in R.$$

In the case when multiplication is well-defined (i.e. the product of two such finite sums is a finite sum again) this set is a ring called a *(left) skew (or twisted) Laurent polynomial ring*, and is denoted by $R[X, X^{-1}; \sigma, \delta]$.

In their interesting recent paper Greenfeld, Smoktunowicz and Ziembowski asked the following questions [4, Question 6.6]: Is there R with no non-zero

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