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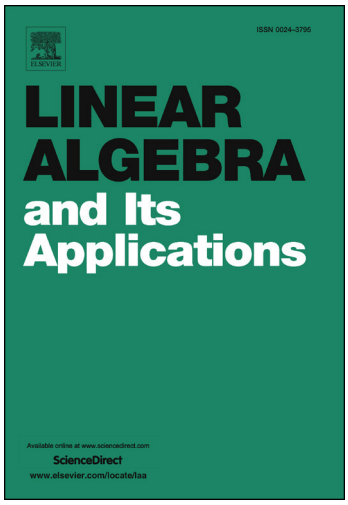

Multiplicatively local spectrum-preserving maps

Abdellatif Bourhim, Ji Eun Lee

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MULTIPLICATIVELY LOCAL SPECTRUM-PRESERVING MAPS

ABDELLATIF BOURHIM AND JI EUN LEE

ABSTRACT. Let X and Y be two infinite-dimensional complex Banach spaces, and $\mathscr{B}(X)$ (resp. $\mathscr{B}(Y)$) be the algebra of all bounded linear operators on X (resp. on Y). Fix two nonzero vectors $x_0 \in X$ and $y_0 \in Y$, and let $\mathscr{B}_{x_0}(X)$ (resp. $\mathscr{B}_{y_0}(Y)$) be the collection of all operators in $\mathscr{B}(X)$ (resp. in $\mathscr{B}(Y)$) vanishing at x_0 (resp. at y_0). We show that if two maps φ_1 and φ_2 from $\mathscr{B}(X)$ onto $\mathscr{B}(Y)$ satisfy

$$\sigma_{\varphi_1(S)\varphi_2(T)}(y_0) = \sigma_{ST}(x_0), \qquad (S, T \in \mathscr{B}(X)),$$

then φ_2 maps $\mathscr{B}_{x_0}(X)$ onto $\mathscr{B}_{y_0}(Y)$ and there exist two bijective linear mappings $A : X \to Y$ and $B : Y \to X$ such that $Ax_0 = y_0$, and $\varphi_1(T) = ATB$ for all $T \in \mathscr{B}(X)$ and $\varphi_2(T) = B^{-1}TA^{-1}$ for all $T \notin \mathscr{B}_{x_0}(X)$. When $X = Y = \mathbb{C}^n$, we show that the surjectivity condition on φ_1 and φ_2 is redundant. Furthermore, some known results are obtained as immediate consequences of our main results.

1. INTRODUCTION

In recent years, there has been considerable interest in studying *nonlinear preserver problems*. These problems involve maps between algebras that leave invariant certain properties or subsets or relations without assuming any algebraic condition like linearity or additivity or multiplicativity. The first result of this kind is due to Kowalski and Słodkowski [32] and dates back to 1980. It generalizes the well-known theorem of Gleason-Kahane-Żelazko in the theory of Banach algebra [29, 31] that states that every unital invertibility preserving linear map from a Banach algebra to a semisimple commutative Banach algebra is multiplicative.

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Key words and phrases. Nonlinear preservers, Local spectrum, the single-valued extension property, finite rank operators.

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