

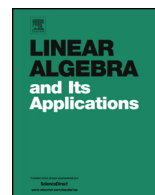


ELSEVIER

Contents lists available at ScienceDirect

# Linear Algebra and its Applications

[www.elsevier.com/locate/laa](http://www.elsevier.com/locate/laa)



## A family of graphs that are determined by their normalized Laplacian spectra <sup>☆</sup>

Abraham Berman <sup>a</sup>, Dong-Mei Chen <sup>b</sup>, Zhi-Bing Chen <sup>b</sup>,  
Wen-Zhe Liang <sup>c</sup>, Xiao-Dong Zhang <sup>c,\*</sup>

<sup>a</sup> Department of Mathematics, Technion-Israel Institute of Technology, Haifa 32000, Israel

<sup>b</sup> College of Mathematics and Statistics, Shenzhen University, Shenzhen 518060, PR China

<sup>c</sup> School of Mathematical Sciences, MOE-LSC, SHL-MAC, Shanghai Jiao Tong University, Shanghai 200240, PR China

### ARTICLE INFO

#### Article history:

Received 1 November 2017

Accepted 1 March 2018

Available online xxx

Submitted by R. Brualdi

#### MSC:

05C50

05C35

#### Keywords:

Normalized Laplacian spectrum

Generalized friendship graph

Spectral characterization

Cospectral graph

### ABSTRACT

Let  $F_{p,q}$  be the generalized friendship graph  $K_1 \vee (pK_q)$  on  $pq + 1$  vertices obtained by joining a vertex to all vertices of  $p$  disjoint copies of the complete graph  $K_q$  on  $q$  vertices. In this paper, we prove that  $F_{p,q}$  is determined by its normalized Laplacian spectrum if and only if  $q \geq 2$ , or  $q = 1$  and  $p \leq 2$ .

© 2018 Elsevier Inc. All rights reserved.

<sup>☆</sup> This work is supported by the Joint NSFC-ISF Research Program (jointly funded by the National Natural Science Foundation of China and the Israel Science Foundation (Nos. 11561141001, 2219/15), the National Natural Science Foundation of China (No. 11531001).

\* Corresponding author.

E-mail address: [xiaodong@sjtu.edu.cn](mailto:xiaodong@sjtu.edu.cn) (X.-D. Zhang).

### 1. Introduction

Spectral graph theory studies the relations between the structure of a graph and eigenvalues of matrices associated with it. One of the main problems in spectral graph theory is which graphs are determined by their spectrum or equivalently, finding nonisomorphic graphs  $G$  and  $H$  that have the same spectrum. Many results on these questions can be found in two excellent surveys (see [9,10]) by Van Dam and Haemers.

Let  $G$  be an undirected simple graph with vertex set  $V(G)$  and edge set  $E(G)$ . Let  $d_v$  be the degree of a vertex  $v \in V(G)$ . The *normalized Laplacian matrix* of a graph  $G$  is defined to be  $\mathcal{L}(G) = (l_{uv})$ , where

$$l_{uv} = \begin{cases} 1, & \text{if } u = v \text{ and } u \text{ is not an isolated vertex;} \\ -\frac{1}{\sqrt{d_u d_v}}, & \text{if } u \text{ is adjacent to } v; \\ 0, & \text{otherwise.} \end{cases}$$

The eigenvalues of  $\mathcal{L}(G)$  are called  $\mathcal{L}$ -eigenvalues. This paper deals with the normalized Laplacian matrix of  $G$ , so we denote its spectrum of  $G$  (the all eigenvalues  $\mathcal{L}(G)$  of  $G$ , including multiplicities) by  $Sp(G)$ . We say that  $G$  and  $H$  are *cospectral* if they are not isomorphic, but  $Sp(G) = Sp(H)$ , and that  $G$  is determined by its normalized Laplacian spectrum if  $Sp(H) = Sp(G)$  only when  $H$  is isomorphic to  $G$ .

Chung in [4] showed how the normalized Laplacian spectrum reveals fundamental properties and structure of a graph. Butler [1] surveyed algebraic aspects of  $\mathcal{L}(G)$  and provided (see [2] and [3]) several methods of constructing cospectral graphs. Almost all small graphs are determined by their normalized Laplacian spectrum (see [3]). The normalized Laplacian spectrum of a complete graph  $K_n$  is 0,  $\frac{n}{n-1}$  with multiplicities 1 and  $n - 1$  respectively, and  $K_n$  is determined by this spectrum [3].

Butler [1] conjectured that the only graphs cospectral with a cycle are  $K_{1,3}$  and the graph  $\gamma_{4k}$  obtained by identifying the center vertex of a path on  $2k + 1$  vertices and a vertex of a cycle on  $2k$  vertices; i.e., a cycle on  $n$  vertices is determined by its normalized Laplacian spectrum if and only if  $n > 4$  and  $4 \nmid n$ . In general, up to now, there are very few graphs that are known to be determined by their normalized Laplacian spectrum. In this paper, we present a family of graphs that are determined by their spectrum.

Denote by  $F_{p,q}$  the graph  $K_1 \vee (pK_q)$  on  $pq + 1$  vertices obtained by joining a vertex to all vertices of  $p$  disjoint copies of the complete graph  $K_q$  on  $q$  vertices. The *friendship graph*  $F_k$  consists of  $k$  edge-disjoint triangles that meet in one vertex (for the famous friendship theorem, see [6]). Liu et al. [8] proved the  $F_k$  is determined by its Laplacian spectrum (see also [7]). Wang et al. [12] proved that  $F_k$  is determined by its signless Laplacian spectrum. Recently, Cioabă et al. [5] proved that  $F_k$  is determined by its adjacent spectrum if and only if  $k \neq 16$ , and that for  $F_{16}$ , there is only one graph nonisomorphic to  $F_{16}$ , but with the same adjacency spectrum. The friendship graphs are a subfamily of  $F_{p,q}$ , since  $F_k = F_{k,2}$ .

Download English Version:

<https://daneshyari.com/en/article/8897862>

Download Persian Version:

<https://daneshyari.com/article/8897862>

[Daneshyari.com](https://daneshyari.com)