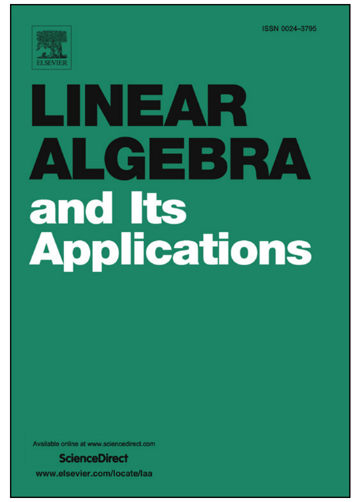
Accepted Manuscript



Colorings beyond Fox: the other linear Alexander quandles

Louis H. Kauffman, Pedro Lopes

PII: S0024-3795(17)30653-5
DOI: https://doi.org/10.1016/j.laa.2017.11.025
Reference: LAA 14392

To appear in: *Linear Algebra and its Applications*

Received date: 6 August 2017
Accepted date: 24 November 2017

Please cite this article in press as: L.H. Kauffman, P. Lopes, Colorings beyond Fox: the other linear Alexander quandles, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2017.11.025

Colorings beyond Fox: the other linear Alexander quandles.

Louis H. Kauffman
Department of Mathematics, Statistics and Computer Science,
University of Illinois at Chicago, 851 S. Morgan St.,
Chicago IL 60607-7045, USA
kauffman@uic.edu
and
Pedro Lopes
Center for Mathematical Analysis, Geometry, and Dynamical Systems,
Department of Mathematics,
Instituto Superior Técnico, Universidade de Lisboa
1049-001 Lisbon, Portugal
pelopes@math.tecnico.ulisboa.pt

December 1, 2017

Abstract

This article is about applications of linear algebra to knot theory. For example, for odd prime p, there is a rule (given in the article) for coloring the arcs of a knot or link diagram from the residues mod p. This is a knot invariant in the sense that if a diagram of the knot under study admits such a coloring, then so does any other diagram of the same knot. This is called p-colorability. It is also associated to systems of linear homogeneous equations over the residues mod p, by regarding the arcs of the diagram as variables and assigning the equation "twice the over-arc minus the sum of the under-arcs equals zero" to each crossing. The knot invariant is here the existence or non-existence of non-trivial solutions of these systems of equations, when working over the integers modulo p (a non-trivial solution is such that not all variables take up the same value). Another knot invariant is the minimum number of distinct colors (values) these non-trivial solutions require, should they exist. This corresponds to finding a basis, supported by a diagram, in which these solutions take up the least number of distinct values. The actual minimum is hard to calculate, in general. For the first few primes, less than 17, it depends only on the prime, p, and not on the specific knots that admit non-trivial solutions, modulo p. For primes larger than 13 this is an open problem. In this article, we begin the exploration of other generalizations of these colorings (which also involve systems of linear homogeneous equations mod p) and we give lower bounds for the number of colors.

Keywords: Linear Alexander quandle; dihedral quandle; Fox coloring; minimum number of colors; crossing number; determinant of knot or link.
Mathematics Subject Classification 2010: 57M25, 57M27

1 Introduction

This article is about the application of linear algebra to the theory of knots and links. Knots and links are embeddings of circles into 3-dimensional space [9]. We will use the word "knot" to mean both "knots" (one component embeddings) and "links" (multiple component embeddings); wherever necessary, we will emphasize that we mean a one component link (or a multiple component link). Knot theorists work with knots by projecting them onto a plane, thereby obtaining the so-called knot diagrams. See Figure 1 for an illustration of a knot diagram. Note that for our purposes, a knot diagram is finite in the sense that it has a finite number of arcs and crossings. Knots obtained by deformation from a given knot are in