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A simplified approach to Fiedler-like pencils via block minimal bases pencils

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A B S T R A C T

The standard way of solving the polynomial eigenvalue problem associated with a matrix polynomial is to embed the matrix coefficients of the polynomial into a matrix pencil, transforming the problem into an equivalent generalized eigenvalue problem. Such pencils are known as linearizations. Many of the families of linearizations for matrix polynomials available in the literature are extensions of the so-called

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family of Fiedler pencils. These families are known as generalized Fiedler pencils, Fiedler pencils with repetition, and generalized Fiedler pencils with repetition—or Fiedler-like pencils for simplicity. The goal of this work is to unify the Fiedler-like pencils approach with the more recent one based on strong block minimal bases pencils introduced in F.M. Dopico et al. (2017) [17]. To this end, we introduce a family of pencils that we have named extended block Kronecker pencils, whose members are, under some generic nonsingularity conditions, strong block minimal bases pencils, and show that, with the exception of the non-proper generalized Fiedler pencils, all Fiedler-like pencils belong to this family modulo permutations. As a consequence of this result, we obtain a much simpler theory for Fiedler-like pencils than the one available so far. Moreover, we expect this simplification to allow for further developments in the theory of Fiedler-like pencils such as global or local backward error analyses and eigenvalue conditioning analyses of polynomial eigenvalue problems solved via Fiedler-like linearizations.

1. Introduction

Matrix polynomials and their associated *polynomial eigenvalue problems* appear in many areas of applied mathematics, and they have received in the last years considerable attention. For example, they are ubiquitous in a wide range of problems in engineering, mechanics, control theory, computer-aided graphic design, etc. For detailed discussions of different applications of matrix polynomials, we refer the reader to the classical references [25,30,45], the modern surveys [3, Chapter 12] and [39,46] (and the references therein), and [35–37]. For those readers not familiar with the theory of matrix polynomials and polynomial eigenvalue problems, those topics are briefly reviewed in Section 2.

The standard way of solving the polynomial eigenvalue problem associated with a matrix polynomial is to *linearize* the polynomial into a matrix pencil (i.e., matrix polynomials of grade 1), known as linearization [15,24,25]. The linearization process transforms the polynomial eigenvalue problem into an equivalent generalized eigenvalue problem, which, then, can be solved using eigensolvers such as the QZ algorithm or the staircase algorithm, in the case of singular matrix polynomials [40,48,49]. Ideally, to make a set of linearizations desirable for numerical applications, it should satisfy the following list of properties:

(i) the linearizations should be *strong linearizations*, regardless whether the matrix polynomial is regular or singular;
(ii) the linearizations should be easily constructible from the coefficients of the matrix polynomials (ideally, without any matrix operation other than scalar multiplication);