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Free-coordinate estimation for doubly multivariate data

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ABSTRACT

The article addresses the best unbiased estimators of the block compound symmetric covariance structure for $m-$variate observations with equal mean vector over u sites under the assumption of multivariate quasi-normality. The free-coordinate approach is used to prove that the quadratic estimation of covariance parameters is equivalent to linear estimation with a properly defined inner product in the space of symmetric matrices. Without assumption of normality but quasi-normality, meaning that up to fourth moments are the same as in the normal case, the estimators are best linear and best quadratic for mean and covariance parameters, respectively. Finally, strong consistency is proven. The properties of the estimators in the proposed model are compared against a similar model available in the literature.

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An application of the proposed approach to a clinical study data is presented.

1. Introduction

Doubly multivariate data or multivariate repeated measures data, where more than one response variable ($m > 1$) is measured on each experimental unit (person, animal, plant...) at more than one site or more than one time point ($u > 1$), has become increasingly noticeable these days in a wide diversity of disciplines, such as biomedical, medical, environmental, pharmaceutical and psychological research. In this article, we consider an example from a clinical study in the prevention of osteoporosis. An investigator measured the mineral content of bones by photon absorptiometry to examine whether dietary supplements would slow the bone loss in 25 elder women. Bone mineral content was measured from each woman at three different anatomic locations ($m = 3$) of the body: radius, humerus and ulna as well as on the dominant and non-dominant sides ($u = 2$). Osteoporosis is an age-related disorder involving in a progressive decrease in bone mass due to the loss of minerals mainly calcium and vitamin D. As a result, bones become weak and brittle, and more susceptible to fractures. In a person with severe osteoporosis, fractures can occur from lifting even light objects, or from falls that would not even bruise or injure the average person. Currently, it is estimated that one in every four post-menopausal women has osteoporosis. Although it is more common in white or Asian women older than 50 years, osteoporosis can occur in almost any person at any age: osteoporosis is not just an 'old woman's disease'. More than two million American men currently have osteoporosis and more than 30 percent of hip fractures occur in men. Osteoporosis is recognized as a major public health issue, with more than two million osteoporosis-related fractures occurring annually in the United States alone [5]. The estimated national cost for osteoporosis and related injuries is $14 billion each year in the United States. Fortunately, we can do several things to ensure that bones are not at risk for these men and women [29]. Nevertheless, to model the data one must take into account the correct structure of the variance-covariance matrix.

Block exchangeable or block compound symmetric (BCS) covariance structure (defined in Section 2) may be a realistic assumption in many doubly multivariate data. Block compound symmetric covariance structure for doubly multivariate data is a multivariate generalization of compound symmetry covariance structure for multivariate data. Multivariate generalization of compound symmetry covariance structure to multiple characters was first introduced by Rao ([20] and [21]), while discriminating genetically different groups. Later BCS structure has been studied most extensively by Arnold [1],