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# Second homology of generalized periplectic Lie superalgebras

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Let $(R, ^-)$ be an arbitrary unital associative superalgebra with superinvolution over a commutative ring $\Bbbk$ with 2 invertible. The second homology of the generalized periplectic Lie superalgebra $\mathfrak{p}_m(R, ^-)$ for $m \geqslant 3$ has been completely determined via an explicit construction of its universal central extension. In particular, this second homology is identified with the first $\mathbb{Z}/2\mathbb{Z}$-graded dihedral homology of $R$ with certain superinvolution whenever $m \geqslant 5$.


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## 1. Introduction

It is well known that the second homology of a Lie (super)algebra $\mathfrak{g}$ is identified with the kernel of its universal central extension, and thus classifies all central extensions of $\mathfrak{g}$ up to isomorphism (cf. [16,18]). They play crucial roles in the theory of Lie (super)algebras.

A remarkable work about the second homology of a Lie algebra is the nice connection between the second homology of a matrix Lie algebra and the first cyclic homology of its coordinates associative algebra established in [10]. Let $A$ be a unital associative algebra over a commutative ring $\Bbbk$ with 2 invertible. One denotes $\mathfrak{gl}_n(A)$ the Lie algebra of all $n \times n$-matrices with entries in $A$ under commutator operation and $\mathfrak{sl}_n(A)$ the derived Lie subalgebra of $\mathfrak{gl}_n(A)$. It is shown in [10] that the second homology $\mathrm{H}_2(\mathfrak{sl}_n(A))$ with $n \geqslant 2$ is isomorphic to the first cyclic homology $\mathrm{HC}_1(A)$. Such an isomorphism has been extended to many other classes of Lie (super)algebras. For instance, Y. Gao showed in [6] that the second homology of elementary unitary Lie algebra $\mathfrak{eu}_n(R, ^-)$ with $n \geqslant 5$ is identified with the first skew-dihedral homology of $(R, ^-)$ that is a unital associative algebra with anti-involution.

In the particular case where the coordinates algebra is associative and commutative, the matrix Lie algebras under consideration are isomorphic to current Lie algebras of the form $\mathfrak{g} \otimes A$, where $\mathfrak{g}$ is a finite dimensional (simple) Lie algebra and $A$ is an associative commutative algebra. For instance, $\mathfrak{sl}_n(A)$ is isomorphic to $\mathfrak{sl}_n(\Bbbk) \otimes_{\Bbbk} A$ if $A$ is commutative, while $\mathfrak{eu}_n(R, \mathrm{id})$ is isomorphic to $\mathfrak{eu}_n(\Bbbk, \mathrm{id}) \otimes_{\Bbbk} R$ if $R$ is commutative. Low-degree (co)homology of current Lie algebras were also studied intensively. In [15], K.-H. Neeb and F. Wagemann explicitly described the second cohomology of current algebras of general Lie algebras with coefficients in a trivial module. More details could be found in [15] and the references therein.

The super analogue of C. Kassel and J.L. Loday's work was obtained in [3,4]. The isomorphism between the second homology of the Lie superalgebra $\mathfrak{sl}_{m|n}(S)$ coordinated by a unital associative superalgebra $S$ with $m + n \geqslant 5$ and the first $\mathbb{Z}/2\mathbb{Z}$-graded cyclic homology $\mathrm{HC}_1(S)$ was established. Recent investigation [2] further gave the identification between the second homology of the ortho-symplectic Lie superalgebra $\mathfrak{osp}_{m|2n}(R, ^-)$ and the first $\mathbb{Z}/2\mathbb{Z}$-graded skew-dihedral homology of $(R, ^-)$ for $(m, n) \neq (1, 1)$ or $(2, 1)$, where $(R, ^-)$ is a unital associative superalgebra with superinvolution (see (2.1) for the definition). A series of deep investigations on the relationship between the homology theory of Lie algebras and the homology theory of associative algebras have been made in [12,13].

Inspired by the above developments, we aim to establish an isomorphism that is analogous to C. Kassel and J.L. Loday's isomorphism for the generalized periplectic Lie superalgebra $\mathfrak{p}_m(R, ^-)$ coordinatized by a unital associative superalgebra $(R, ^-)$ with superinvolution. As in Section 2, a generalized periplectic Lie superalgebra is defined as the derived sub-superalgebra of the Lie superalgebra of all skew-symmetric matrices with respect to the so-called periplectic superinvolution. It is a super analogue of a unitary Lie