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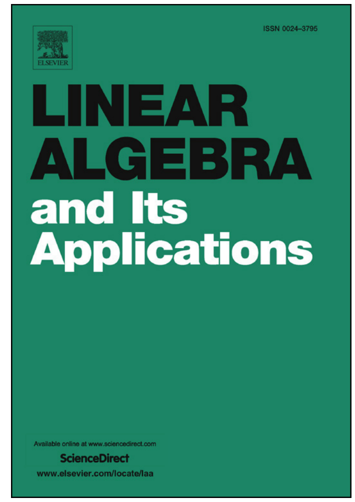

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NEW CHARACTERIZATIONS OF OPERATOR MONOTONE FUNCTIONS

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Abstract. If σ is a symmetric mean and f is an operator monotone function on $[0,\infty)$, then

$$f(2(A^{-1}+B^{-1})^{-1}) \leq f(A\sigma B) \leq f((A+B)/2).$$

Conversely, Ando and Hiai showed that if f is a function that satisfies either one of these inequalities for all positive operators A and B and a symmetric mean different than the arithmetic and the harmonic mean, then the function is operator monotone.

In this paper, we show that the arithmetic and the harmonic means can be replaced by the geometric mean to obtain similar characterizations. Moreover, we give characterizations of operator monotone functions using self-adjoint means and general means subject to a constraint due to Kubo and Ando.

1. Introduction

It is well-known that if σ is a symmetric mean of operators, *i.e.*, $A\sigma B = B\sigma A$, the following inequality is satisfied for any positive operators A and B,

$$A!B \leq A\sigma B \leq A\nabla B, \tag{1}$$

where $A!B = 2(A^{-1}+B^{-1})^{-1}$ is the harmonic mean of A and B, and $A\nabla B = (A+B)/2$ is the arithmetic mean of A and B. Obviously, if $f : [0,\infty) \to [0,\infty)$ is operator monotone, we have

$$f(A!B) \leq f(A\sigma B) \leq f(A\nabla B). \tag{2}$$

Interestingly, if a continuous function f satisfies either of the inequalities for some scalar mean M,

$$f(a!b) \leq f(M(a,b)) \leq f(a\nabla b). \tag{3}$$

for positive numbers a and b, then f is monotone increasing. Matrix generalizations of this observation for Kubo-Ando means were discussed by Hiai and Ando in [1,

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