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The normalized numerical range and the Davis-Wielandt shell

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Abstract

For a given n -by- n matrix A , its *normalized numerical range* $F_N(A)$ is defined as the range of the function $f_{N,A}: x \mapsto (x^*Ax)/(\|Ax\|\cdot\|x\|)$ on the complement of $\ker A$. We provide an explicit description of this set for the case when A is normal or $n = 2$. This extension of earlier results for particular cases of 2-by-2 matrices (by Gevorgyan) and essentially Hermitian matrices of arbitrary size (by A. Stoica and one of the authors) was achieved due to the fresh point of view at $F_N(A)$ as the image of the Davis-Wielandt shell $DW(A)$ under a certain non-linear mapping $h: \mathbb{R}^3 \mapsto \mathbb{C}$.

Keywords: Normalized numerical range, Davis-Wielandt shell, normal matrix

2010 MSC: 15A60 47A12 47B15

1. Introduction

Throughout the paper, we denote by \mathbb{C}^n the standard n -dimensional inner product space over the complex field \mathbb{C} and by $M_n(\mathbb{C})$ the algebra of all n -by- n matrices with entries in \mathbb{C} .

The classical *numerical range* $F(A)$ (a.k.a. the *field of values*, or the *Hausdorff set*) of $A \in M_n(\mathbb{C})$ is by definition the set of values of the corresponding quadratic form x^*Ax on the unit sphere $S\mathbb{C}^n := \{x \in \mathbb{C}^n : \|x\| = 1\}$ of \mathbb{C}^n . Equivalently,

$$F(A) = \left\{ (x^*Ax)/\|x\|^2 : x \in \mathbb{C}^n \setminus \{0\} \right\}.$$

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