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Sharp upper bounds on the distance energies of a graph

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ABSTRACT

In this paper, for a simple undirected connected graph, sharp upper bounds on the distance energy, distance Laplacian energy and distance signless Laplacian energy are obtained. The graphs attaining the corresponding upper bound are characterized.

1. Introduction

Let $G = (V(G), E(G))$ be a simple undirected graph on n vertices with vertex set $V(G) = \{v_1, \ldots, v_n\}$ and edge set $E(G)$. Let $D(G)$ be the diagonal matrix of order n whose (i,i)-entry is the degree of the vertex v_i of G and let $A(G)$ be the adjacency matrix

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of G. The matrices $L(G) = D(G) - A(G)$ and $Q(G) = D(G) + A(G)$ are the Laplacian and signless Laplacian matrix of G, respectively. The matrices $L(G)$ and $Q(G)$ are both positive semidefinite and $(0, \mathbf{1})$ is an eigenpair of $L(G)$ where $\mathbf{1}$ is the all ones vector.

Let

$$\lambda_1(G) \geq \lambda_2(G) \geq \ldots \geq \lambda_n(G)$$

be the eigenvalues of $A(G)$. They are called the eigenvalues of G. Let

$$\mu_1(G) \geq \mu_2(G) \geq \ldots \geq \mu_n(G) = 0$$

and

$$q_1(G) \geq q_2(G) \geq \ldots \geq q_n(G)$$

be the eigenvalues of $L(G)$ and $Q(G)$, respectively. They are called the Laplacian eigenvalues of G and the signless Laplacian eigenvalues of G, respectively. It is well known that the Laplacian eigenvalues and signless Laplacian eigenvalues of G coincide if and only if G is a bipartite graph.

The Frobenius norm of an $n \times n$ matrix $M = (m_{i,j})$ is

$$\|M\|_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} |m_{i,j}|^2}.$$

We recall if M is a normal matrix then $\|M\|_F^2 = \sum_{i=1}^{n} |\lambda_i(M)|^2$ where $\lambda_1(M), \ldots, \lambda_n(M)$ are the eigenvalues of M.

Throughout this paper, we assume that G is a connected graph of order n and K_n denotes the complete graph on n vertices.

The distance between $u, v \in V(G)$ for a connected graph G, denoted by $d(u, v)$, is the length of the shortest path connecting u and v. The Wiener index $W(G)$ of the graph G is

$$W(G) = \frac{1}{2} \sum_{u,v \in V(G)} d(u, v)$$

and the transmission $Tr(v)$ of a vertex $v \in V(G)$ is the sum of the distances from v to all other vertices of G, that is,

$$Tr(v) = \sum_{u \in V(G)} d(v, u).$$

The graph G is said to be *k-transmission regular* if $Tr(v) = k$ for each vertex $v \in V(G)$. The *distance matrix* $\mathcal{D}(G) = (d_{i,j})$ of G is an $n \times n$ matrix indexed by the vertices