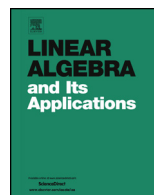




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Sharp upper bounds on the distance energies of a graph



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ABSTRACT

In this paper, for a simple undirected connected graph, sharp upper bounds on the distance energy, distance Laplacian energy and distance signless Laplacian energy are obtained. The graphs attaining the corresponding upper bound are characterized.

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1. Introduction

Let $G = (V(G), E(G))$ be a simple undirected graph on n vertices with vertex set $V(G) = \{v_1, \dots, v_n\}$ and edge set $E(G)$. Let $D(G)$ be the diagonal matrix of order n whose (i, i) -entry is the degree of the vertex v_i of G and let $A(G)$ be the adjacency matrix

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of G . The matrices $L(G) = D(G) - A(G)$ and $Q(G) = D(G) + A(G)$ are the Laplacian and signless Laplacian matrix of G , respectively. The matrices $L(G)$ and $Q(G)$ are both positive semidefinite and $(0, \mathbf{1})$ is an eigenpair of $L(G)$ where $\mathbf{1}$ is the all ones vector.

Let

$$\lambda_1(G) \geq \lambda_2(G) \geq \dots \geq \lambda_n(G)$$

be the eigenvalues of $A(G)$. They are called the eigenvalues of G . Let

$$\mu_1(G) \geq \mu_2(G) \geq \dots \geq \mu_n(G) = 0$$

and

$$q_1(G) \geq q_2(G) \geq \dots \geq q_n(G)$$

be the eigenvalues of $L(G)$ and $Q(G)$, respectively. They are called the Laplacian eigenvalues of G and the signless Laplacian eigenvalues of G , respectively. It is well known that the Laplacian eigenvalues and signless Laplacian eigenvalues of G coincide if and only if G is a bipartite graph.

The Frobenius norm of an $n \times n$ matrix $M = (m_{i,j})$ is

$$\|M\|_F = \sqrt{\sum_{i=1}^n \sum_{j=1}^n |m_{i,j}|^2}.$$

We recall if M is a normal matrix then $\|M\|_F^2 = \sum_{i=1}^n |\lambda_i(M)|^2$ where $\lambda_1(M), \dots, \lambda_n(M)$ are the eigenvalues of M .

Throughout this paper, we assume that G is a connected graph of order n and K_n denotes the complete graph on n vertices.

The distance between $u, v \in V(G)$ for a connected graph G , denoted by $d(u, v)$, is the length of the shortest path connecting u and v . The Wiener index $W(G)$ of the graph G is

$$W(G) = \frac{1}{2} \sum_{u,v \in V(G)} d(u, v)$$

and the transmission $Tr(v)$ of a vertex $v \in V(G)$ is the sum of the distances from v to all other vertices of G , that is,

$$Tr(v) = \sum_{u \in V(G)} d(v, u).$$

The graph G is said to be k -transmission regular if $Tr(v) = k$ for each vertex $v \in V(G)$. The distance matrix $\mathcal{D}(G) = (d_{i,j})$ of G is an $n \times n$ matrix indexed by the vertices

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