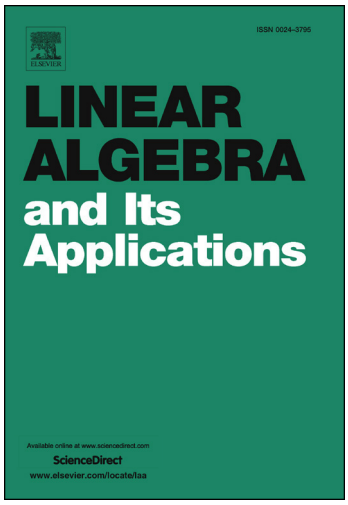
Accepted Manuscript



Threshold incomplete factorization constraint preconditioners for saddle-point matrices

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PII: S0024-3795(18)30048-X
DOI: https://doi.org/10.1016/j.laa.2018.01.034
Reference: LAA 14458

To appear in: *Linear Algebra and its Applications*

Received date: 1 June 2017
Accepted date: 25 January 2018

Please cite this article in press as: S. Lungten et al., Threshold incomplete factorization constraint preconditioners for saddle-point matrices, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2018.01.034

Threshold incomplete factorization constraint preconditioners for saddle-point matrices[☆]

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Abstract

This paper presents a drop-threshold incomplete $\mathrm{LD}^{-1}\mathrm{L}^{\mathrm{T}}$ (δ) factorization constraint preconditioner for saddle-point systems using a threshold parameter δ. A transformed saddle-point matrix is partitioned into a block structure with blocks of order 1 and 2 constituting 'a priori pivots'. Based on these pivots an incomplete $\mathrm{LD}^{-1}\mathrm{L}^{\mathrm{T}}$ (δ) factorization constraint preconditioner is computed that approaches an exact form as δ approaches zero. We prove that both the exact and incomplete factorizations exist such that the entries of the constraint block remain unaltered in the triangular factors. Numerical results are presented for validation.

Keywords: Saddle-point matrices, Transformation, Incomplete factorization, Constraint preconditioner
MSC: 15A23, 65F05, 65F50

1. Introduction

We focus on sparse and large saddle-point systems

$$\begin{aligned} &\mathcal{M}u = b, \\ &\text{with } \mathcal{M} = \mathcal{M}(A) = \begin{bmatrix} A & B^T \\ B & 0 \end{bmatrix},\ u = \begin{bmatrix} x \\ y \end{bmatrix},\ b = \begin{bmatrix} f \\ g \end{bmatrix}, \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ is a symmetric positive definite (SPD) matrix, $B \in \mathbb{R}^{m\times n}$ (with $m < n$) is a constraint matrix of full rank, and the vectors $x, f \in \mathbb{R}^n$, and $y, g \in \mathbb{R}^m$. Such systems originate for instance from constrained optimization problems [52, pp. 137–276], discretization of Stokes equations [50] and Maxwell equations [53], and network analysis in electronic circuits [54] and water distribution systems [55].

[☆]This work was supported by the Erasmus Mundus IDEAS project.
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