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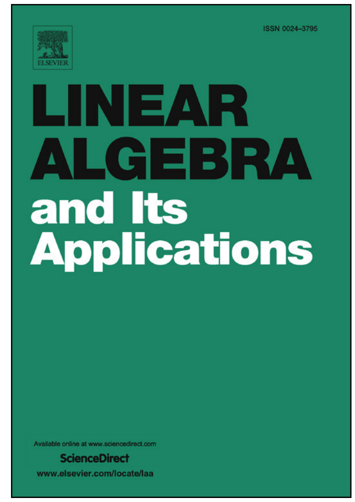

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MULTIPLIERS FOR VON NEUMANN-SCHATTEN BESSEL SEQUENCES IN SEPARABLE BANACH SPACES

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Abstract. In this paper we introduce the concept of von Neumann-Schatten Bessel multipliers in separable Banach spaces and obtain some of their properties. We study their behavior when the symbol belongs to ℓ^p. Also, we investigate the continuous dependency of Hilbert–Schmidt Bessel multipliers on their parameters. Special attention is devoted to the study of invertible Hilbert-Schmidt frame multipliers which is an extensive class of ordinary frame multipliers and includes g-frame multipliers and fusion frame multipliers as elementary examples. Among other things, the invertibility of Hilbert-Schmidt Riesz multipliers is completely characterized. In particular, we show that all the existing frames in Hilbert spaces is uniquely determined by the set of its dual frames.

1. Introduction

Due to the fundamental works done by Feichtinger and his coauthors [29, 30], Fourier and Gabor multipliers were formally introduced and popularized from then on. Fourier and Gabor multipliers play an important role in theory and applications; For more information about the history of this class of operators, some of their properties and their applications in scientific disciplines and in modern life the reader can consult Section 1 of the papers [15, 52] and the references (for examples) [16, 22]. Balazs [8] extended the notion of Gabor multipliers to arbitrary Hilbert space frames. In details, he considered the operators of the form

$$\mathbf{M}_{\mathfrak{m},\Phi,\Psi}(f) = \sum_{i=1}^{\infty} \mathfrak{m}_i \langle f, \psi_i \rangle \varphi_i \qquad (f \in \mathbb{H}), \tag{1.1}$$

where $\Phi = \{\varphi_i\}_{i=1}^{\infty}$ and $\Psi = \{\psi_i\}_{i=1}^{\infty}$ are ordinary Bessel sequences in the Hilbert space $\mathbb{H}$, and $\mathfrak{m} = \{\mathfrak{m}_i\}_{i=1}^{\infty}$ is a bounded complex scalar sequence in $\mathbb{C}$. It is worthwhile to mention that this class of operators is not only of interest for applications in modern life (see for example [2, 14, 40, 41]), but also it is of utmost importance in different branches of linear algebra, matrix analysis and functional analysis. For example, they are used for the diagonalization of matrices [33, Definition 3.1], the diagonalization of operators [9, 23, 49], matrix representation of operators using frames [10], Galerkin–like representation of operators [13] and for

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