# Signature of power graphs 

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## A R T I C L E I N F O

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The signature $s(G)$ of a graph $G$ is defined as the difference between its positive inertia index and the negative inertia index. In 2013, H. Ma et al. (2013) [8] conjectured that $-c_{3}(G) \leq s(G) \leq c_{5}(G)$ for an arbitrary simple graph $G$, where $c_{i}(G)$ denotes the number of cycles in $G$ with length $i$ modulo 4. In 2014, L. Wang et al. [10] proved that $-c_{3}\left(T^{k}\right) \leq$ $s\left(T^{k}\right) \leq c_{5}\left(T^{k}\right)$ for any tree $T$ and for any $k \geq 2$. In this paper, we prove that $-c_{3}\left(G^{k}\right) \leq s\left(G^{k}\right) \leq c_{5}\left(\bar{G}^{k}\right)$ for any simple graph $G$ and for any $k \geq 2$, thus extend the main result of [10] to more general cases.
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## 1. Introduction

Throughout this paper we consider simple graphs, i.e., undirected graphs without loops and multiple edges. Let $G$ be a simple graph of order $n$ with vertex set $V(G)$ and edge set $E(G)$. The adjacency matrix $A(G)$ of $G$ is an $n \times n$ symmetric matrix $\left[a_{x y}\right]$ such that $a_{x y}=1$ if $x$ and $y$ are adjacent and $a_{x y}=0$, otherwise. Since $A(G)$ is a symmetric matrix, its eigenvalues are all real values. The positive (resp., negative) inertia index of

[^0]$G$ is defined as the number of positive (resp., negative) eigenvalues of $A(G)$, which is denoted by $p(G)$ (resp., $n(G)$ ). The signature $s(G)$ of $G$ is defined to be the difference between the positive and negative inertia indices, namely, $s(G)=p(G)-n(G)$.

If $x, y \in V(G)$ are adjacent in $G$ we write $x \sim y$ and denote by $x y$ the edge joining them. The neighbor set of $v \in V(G)$ in $G$ is defined and denoted as $N_{G}(v)=\{u \mid$ $u v \in E(G)\}$, and the degree $d_{G}(v)$ of $v$ in $G$ is defined as $d_{G}(v)=\left|N_{G}(v)\right|$. The minimum degree of $G$, denoted as $\delta(G)$, is the least value among all $d_{G}(v)$ for $v \in V(G)$. We call $v$ a $k$-vertex in $G$ if $d_{G}(v)=k$, and call $v$ a $k^{+}$-vertex if $d_{G}(v) \geq k$. The distance of two vertices $x, y$ in $G$, denoted by $\operatorname{dist}_{G}(x, y)$, is the length of a shortest path between $x$ and $y$ and if no paths between $x$ and $y$ we set $\operatorname{dist}_{G}(x, y)=+\infty$.

For a subset $S$ of $V(G)$ we denote by $G-S$ the induced subgraph of $G$ obtained from $G$ by deleting all vertices in $S$ together with the edges incident to vertices of $S$. For a subset $E$ of $E(G)$, we denote by $G-E$ the spanning subgraph of $G$ obtained by deleting all edges of $E$ from $G$. As usual, we denote by $K_{n}$ (resp., $P_{n}, C_{n}$ ) the complete graph (resp., path, cycle) of order $n$.

In 2013, H. Ma et al. [8] posed the following conjecture on the signature of graphs in terms of the number of odd cycles. Let $c_{i}(G)$ be the number in $G$ with length $i$ module 4. In other words, each cycle in $c_{i}(G)$ has length $4 k+i$. In order to make sense for all the positive integers for $k$, we denote $c_{5}(G)$ instead of $c_{1}(G)$.

Conjecture 1.1. [8] Let $G$ be a simple graph and $s(G)$ be its signature, then

$$
-c_{3}(G) \leq s(G) \leq c_{5}(G)
$$

Since bipartite graphs always have the same positive and negative inertia index (see [2]), the above conjecture clearly holds for bipartite graphs. In the same paper [8], the authors proved the conjecture is true for unicyclic and bicyclic graphs. Y. Jiang ([6]) confirmed the conjecture for simple graphs with edge-disjoint cycles. Recently, D. Wang et al. [9] extended the results for simple graphs with vertex-disjoint cycles. L. Wang et al. [10] confirmed the conjecture for all line graphs and for the power graphs of trees.

For $k \geq 1$, the $k$-th power graph $G^{k}$ of $G$ is obtained from $G$ by adding an edge between every pair of non-adjacent vertices at distance $k$ or less. It is easy to see that $G^{1}=G$ and $G^{d}=K_{n}$ if $d$ is the diameter of $G$. When $k=2$, we call $G^{k}$ the square of $G$. The $k$-th power graph has been found useful. For example, graph coloring on the square of a graph may be used to assign frequencies to the participants of wireless communication networks so that no two participants interfere with each other at any of their common neighbors, and to find graph drawings with high angular resolution ([1,5]). If $G$ is connected, then $G^{3}$ necessarily contains a Hamiltonian cycle. And $G^{2}$ is always Hamiltonian when $G$ is 2-vertex connected ([3]).

For Conjecture 1.1, the authors in [10] gave a concise affirmation for all $k$-th power of trees. That is,

$$
-c_{3}\left(T^{k}\right) \leq s\left(T^{k}\right) \leq c_{5}\left(T^{k}\right)
$$

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