Linear Algebra and its Applications 545 (2018) 240–255

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Further applications of the Cauchon algorithm to rank determination and bidiagonal factorization

Mohammad Adm [a,b,d,*], Khawla Al Muhtaseb [a], Ayed Abedel Ghani [a], Shaun Fallat [d], Jürgen Garloff [b,c]

[a] *Department of Applied Mathematics and Physics, Palestine Polytechnic University, Hebron, Palestine*
[b] *Department of Mathematics and Statistics, University of Konstanz, D-78464 Konstanz, Germany*
[c] *Institute of Applied Research, University of Applied Sciences / HTWG Konstanz, D-78405 Konstanz, Germany*
[d] *Department of Mathematics and Statistics, University of Regina, Regina, SK, S4S 0A2 Canada*

ARTICLE INFO

Article history:
Received 16 June 2017
Accepted 25 January 2018
Available online 2 February 2018
Submitted by J.F. Queiro

MSC:
15A03
15A23
15B99

Keywords:
Rank
Cauchon matrix
Cauchon diagram
Cauchon algorithm

ABSTRACT

For a class of matrices connected with Cauchon diagrams, Cauchon matrices, and the Cauchon algorithm, a method for determining the rank, and for checking a set of consecutive row (or column) vectors for linear independence is presented. Cauchon diagrams are also linked to the elementary bidiagonal factorization of a matrix and to certain types of rank conditions associated with submatrices called descending rank conditions.

* Corresponding author at: Department of Mathematics and Statistics, University of Konstanz, D-78464 Konstanz, Germany.

E-mail addresses: mjamathe@yahoo.com (M. Adm), khawla@ppu.edu (K. Al Muhtaseb), ayed42@ppu.edu (A.A. Ghani), shaun.fallat@uregina.ca (S. Fallat), juergen.garloff@htwg-konstanz.de (J. Garloff).

https://doi.org/10.1016/j.laa.2018.01.035

Linear independence
Elementary bidiagonal factorization

1. Introduction

A fundamental problem in mathematics and its applications is the determination of the rank of a matrix. The rank of a given real n-by-m matrix can be determined by, for example, Gaussian elimination. In this paper, we present an alternative approach to calculating the rank and bidiagonal factorization for matrices that admit a Cauchon matrix upon application of the Cauchon algorithm. This method relies on Cauchon diagrams, Cauchon matrices, and the so-called (condensed) Cauchon algorithm. Like Gaussian elimination, single computations of the Cauchon algorithm can be represented as the computation of minors of order two. In this way, a series of intermediate matrices is produced. The algorithm culminates with a matrix from which we can easily determine the rank of the original matrix.

This algorithm was applied in [1–4], [9], [11], [13] to problems related to totally nonnegative matrices, that is, matrices having all their minors nonnegative. For properties of these matrices, the reader is referred to the monographs [7] or [12]. In [1], [3], a condensed form of the Cauchon algorithm was derived by which the amount of required arithmetic operations was reduced by one order of magnitude to bring it in line with the complexity of performing conventional Gaussian elimination, viz. $O(n^3)$ for a nonsingular n-by-n matrix. In the first part of our paper, we apply the condensed form of the Cauchon algorithm to matrices that admit a Cauchon matrix upon application of this algorithm in order to determine their ranks and linear independence of sets of consecutive row vectors. The class of matrices that admit a Cauchon matrix upon application of the Cauchon algorithm contains the totally nonnegative matrices [11], totally nonpositive matrices (matrices with all of its minors nonpositive) [4], and matrices satisfying the descending rank conditions, see Section 4.

In the second part of the paper, we link Cauchon diagrams to the factorization of matrices into elementary bidiagonal matrices, namely, one-banded, unit diagonal matrices having at most one nonzero off-diagonal entry, see [7, Chapter 2], [12, Chapter 6]. Elementary bidiagonal factorization itself is linked to certain types of rank conditions associated with submatrices called descending rank conditions [10]; see [8] for an earlier paper discussing these conditions. We directly link Cauchon diagrams (or matrices) to the descending rank conditions hereby reducing the complexity by some orders of magnitude.

We point out that all of our results are valid if $\mathbb{R}$ is replaced by any field of characteristic zero.

The organization of our paper is as follows: In the next section we introduce the notation used in our paper and provide some auxiliary results and the concept of a lacunary sequence which are employed in the subsequent sections. In Section 3 we use lacunary sequences in order to determine the rank of a certain matrix, and to check the linear