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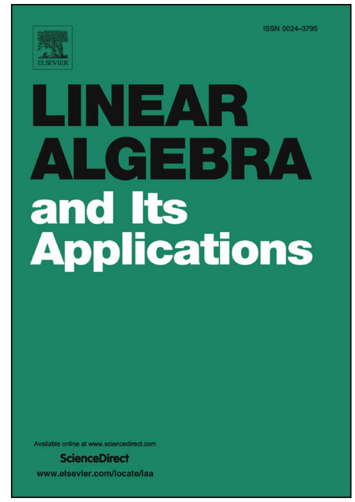

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ON THE SIZE OF SPECIAL FAMILIES OF LINEAR OPERATORS

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Abstract. We continue the study, started in [14], of the search of algebraic structures one can find within the sets of injective linear functions. We shall focus on the cases when the operators are considered both on finite dimensional and infinite dimensional domains. We also study the set of continuous surjective linear operators.

1. Introduction and preliminaries.

In [14], the authors were able to prove that it was not possible to find a vector space of dimension $n+1$ every nonzero element of which is an injective function $f : \mathbb{R}^n \to \mathbb{R}^n$, even though they could construct a vector space of dimension n of such functions. That generalized a result shown in [23] to arbitrary finite dimensions, and it carried out the problem giving some insights concerning infinite dimensions.

In this paper we will have a deeper study concerning the implications of the existence of a vector space contained in the set of injective linear functions between spaces of finite dimensions (plus the zero function). We will also address the case for infinite dimensions and we will introduce the analogous problem for the case of surjective linear functions.

The search of linear structures inside of sets has been a very fruitful field in Mathematics, and in the beginning of the 21^{st} century, some terminology was introduced in an attempt to formalize this idea. More concretely we have the following definition (see, e.g., [1, 2, 5, 23]):

If X is a vector space, μ is a cardinal number and $A \subset X$, then A is said to be *lineable* if there is an infinite dimensional vector space M such that $M \setminus \{0\} \subset A$, and μ*-lineable* if there exists a vector space M with $\dim(M) = \mu$ and $M \setminus \{0\} \subset A$. If, in addition, X is a topological vector space, then A is said to be *spaceable* in X whenever there is a closed infinite dimensional vector subspace M of X satisfying $M \setminus \{0\} \subset A$.

Up to now, a great number of cases have been studied (giving even optimal results, if we want to talk about maximal dimension or cardinality). The monographs [1, 5] provide

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