

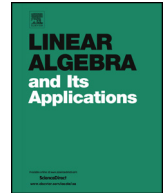


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# Identities with involution for $2 \times 2$ upper triangular matrices algebra over a finite field



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## ABSTRACT

Let  $UT_2(F)$  be the  $2 \times 2$  upper triangular matrices algebra over a finite field  $F$  of characteristic different from 2. For every involution of the first kind of  $UT_2(F)$  we describe the set of all  $*$ -polynomial identities for this algebra.

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## 1. Introduction

Let  $F$  be a field and let  $UT_n(F)$  be the algebra of  $n \times n$  upper triangular matrices over  $F$ . This algebra plays an important role in the theory of PI-algebras. Maltsev was

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the first to describe the set of polynomial identities of  $UT_n(F)$ , denoted by  $Id(UT_n(F))$ . Before to state his result, let  $F\langle X \rangle$  be the free unitary associative algebra freely generated by the infinite set  $X = \{x_1, x_2, \dots\}$ . Maltsev proved in [3] that if the characteristic of  $F$  is  $\text{char}(F) = 0$  then  $Id(UT_n(F))$  is generated, as a T-ideal of  $F\langle X \rangle$ , by the polynomial

$$[x_1, x_2][x_3, x_4] \cdots [x_{2n-1}, x_{2n}],$$

where  $[x, y] = xy - yx$ . The same is hold when  $F$  is infinite. If  $F$  is an arbitrary field (finite or infinite) then

$$Id(UT_n(F)) = [Id(UT_1(F))]^n.$$

This result was proved by Siderov in [4]. Note that  $UT_1(F) = F$ . It's known that  $Id(F)$  is generated as a T-ideal by

- a)  $[x_1, x_2]$  if  $F$  is infinite,
- b)  $[x_1, x_2]$  and  $x_1^q - x_1$  if  $F$  is finite with  $q$  elements.

Using Jacobson's theorem we can omit  $[x_1, x_2]$  in item b). See [2, Exercise 2.3.6] and [1, Theorem 5.78] for details.

Let  $G$  be a group. Valenti and Zaicev described the  $G$ -gradings on  $UT_n(F)$ . They proved in [5] that every  $G$ -grading on  $UT_n(F)$  is elementary. In [6] Vincenzo, Koshlukov and Valenti described the  $G$ -graded polynomial identities of  $UT_n(F)$  if  $F$  is infinite.

Let  $F$  be a field of  $\text{char}(F) \neq 2$ . If  $UT_n(F)$  has an involution, denote by  $Id(UT_n(F), *)$  its  $*$ -polynomial identities. Di Vincenzo, Koshlukov and La Scala described the involutions of first kind of  $UT_n(F)$ . They proved in [7] that there exist two classes of inequivalent involutions when  $n$  is even and a single class otherwise. They also described:

- a)  $Id(UT_2(F), *)$  when  $F$  is infinite,
- b)  $Id(UT_3(F), *)$  when  $\text{char}(F) = 0$ ,

for all involutions of first kind on  $UT_2(F)$  and  $UT_3(F)$  respectively. It's an open problem describe  $Id(UT_n(F), *)$  in other cases.

In this paper we describe  $Id(UT_2(F), *)$  for all involutions of first kind of  $UT_2(F)$  when  $F$  is finite.

## 2. Some properties of finite fields

We will use several times the next two lemmas:

**Lemma 2.1.** [2, Proposition 4.2.3] *Let  $F$  be a finite field,  $|F| = q$ . Then the monomials*

$$x_1^{d_1} \cdots x_n^{d_n}, \quad 0 \leq d_1, \dots, d_n \leq q-1,$$

*are linearly independent modulo  $Id(F)$ .*

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