

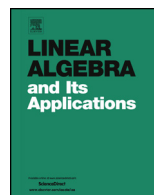


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Principal eigenvectors and spectral radii of uniform hypergraphs

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ABSTRACT

In this paper, some inequalities among the principal eigenvector, spectral radius and vertex degrees of a connected uniform hypergraph are established. Necessary and sufficient conditions of equalities holding are presented, which are related to the regularity of a hypergraph. Furthermore, we present some bounds on the spectral radius for a connected irregular uniform hypergraph in terms of some parameters, such as principal ratio, maximum degree, diameter, and the number of vertices and edges.

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1. Introduction

For a positive integer n , let $[n] = \{1, 2, \dots, n\}$. An order m dimension n tensor $\mathcal{A} = (a_{i_1 i_2 \dots i_m})$ is a multidimensional array with n^m entries, where $i_j \in [n]$, $j \in [m]$. When $m = 2$, \mathcal{A} is an $n \times n$ matrix. Let $\mathbb{C}^{[m,n]}$ be the set of order m dimension n tensors

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over the complex field \mathbb{C} , and \mathbb{C}^n be the set of n -vectors over the complex field \mathbb{C} . For $\mathcal{A} = (a_{i_1 i_2 \dots i_m}) \in \mathbb{C}^{[m,n]}$, if all the entries $a_{i_1 i_2 \dots i_m} \geq 0$, then \mathcal{A} is called nonnegative.

In 2005, Qi [22] and Lim [14] defined the eigenvalues of tensors, respectively. For $\mathcal{A} = (a_{i_1 i_2 \dots i_m}) \in \mathbb{C}^{[m,n]}$ and $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{C}^n$, $\mathcal{A}x^{m-1}$ is an n -vector whose the i -th component is

$$(\mathcal{A}x^{m-1})_i = \sum_{i_2, \dots, i_m=1}^n a_{i i_2 \dots i_m} x_{i_2} \cdots x_{i_m}.$$

If there exists a number $\lambda \in \mathbb{C}$ and a nonzero vector $x = (x_1, \dots, x_n)^T \in \mathbb{C}^n$ such that

$$\mathcal{A}x^{m-1} = \lambda x^{[m-1]},$$

then λ is called an eigenvalue of \mathcal{A} , x is called an eigenvector of \mathcal{A} corresponding to λ , where $x^{[m-1]} = (x_1^{m-1}, x_2^{m-1}, \dots, x_n^{m-1})^T$. The spectral radius $\rho(\mathcal{A}) = \max\{|\lambda| : \lambda \in \sigma(\mathcal{A})\}$, where $\sigma(\mathcal{A})$ is the set of all eigenvalues of \mathcal{A} .

A hypergraph G is a pair $(V(G), E(G))$, where $E(G) \subseteq P(V(G))$ and $P(V(G))$ stands for the power set of $V(G)$. The elements of $V(G)$ are called the vertices and the elements of $E(G)$ are called the edges (see [1]). If each edge of G contains exactly k distinct vertices, then G is called k -uniform. When $k = 2$, G is a graph. For all $i \in V(G)$, $E_i(G)$ denotes the set of edges containing i , and $d_i = |E_i(G)|$ denotes the degree of i , $\Delta = \max_i \{d_i\}$ and $\delta = \min_i \{d_i\}$. If $\Delta = \delta$, then G is called regular. The adjacency tensor [6] of a k -uniform hypergraph G , denoted by \mathcal{A}_G , is an order k dimension $|V(G)|$ nonnegative tensor with entries

$$a_{i_1 i_2 \dots i_k} = \begin{cases} \frac{1}{(k-1)!}, & \text{if } \{i_1, i_2, \dots, i_k\} \in E(G), \\ 0, & \text{otherwise.} \end{cases}$$

Eigenvalues of \mathcal{A}_G are called eigenvalues of G , the spectral radius of \mathcal{A}_G is called the spectral radius of G , denoted by $\rho(G)$.

For k -uniform hypergraph G with n vertices, it is connected if and only if \mathcal{A}_G is nonnegative weakly irreducible [8,21,27]. By the Perron–Frobenius theorem [27], $\rho(G)$ is an eigenvalue of G and there exists a unique positive eigenvector $x = (x_1, \dots, x_n)^T$ corresponding to $\rho(G)$ with $\sum_{i=1}^n x_i^k = 1$, which is called the principal eigenvector of G . The maximum and minimum entries of x are denoted by x_{\max} and x_{\min} , respectively. The parameter $\gamma = \frac{x_{\max}}{x_{\min}}$ is called the principal ratio of G (see [17]). For $e = \{i_1, i_2, \dots, i_k\} \in E(G)$, let $x^e = \prod_{j=1}^k x_{i_j}$ and $x^{e \setminus \{i_1\}} = \prod_{j=2}^k x_{i_j}$.

The principal eigenvector of a connected graph is one of centrality metrics [11]. Let G be a connected irregular graph. Investigations have been conducted to the relationship between x_{\max} and the structure of a connected graph [20], bounds on γ , x_{\max} , x_{\min}

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