

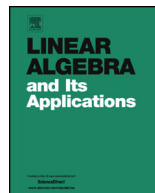


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Signless Laplacian energy of a graph and energy of a line graph [☆]

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ABSTRACT

For a simple graph G of order n , size m and with signless Laplacian eigenvalues $q_1, q_2, \dots, q_n$, the signless Laplacian energy $QE(G)$ is defined as $QE(G) = \sum_{i=1}^n |q_i - \bar{d}|$, where $\bar{d} = \frac{2m}{n}$ is the average vertex degree of G . We obtain the lower bounds for $QE(G)$, in terms of first Zagreb index $M_1(G)$, maximum degree d_1 , second maximum degree d_2 , minimum degree d_n and second minimum degree d_{n-1} . As a consequence of these bounds, we obtain several bounds for the energy $E(\mathcal{L}(G))$ of the line graph $\mathcal{L}(G)$ of graph G in terms of various graph parameters like $M_1(G)$, ω (the clique number), n , m , etc., which improve some recently known bounds.

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1. Introduction

Let $G(V, E)$ be a simple graph with n vertices, m edges with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \dots, e_m\}$. In case the graph G is to be

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mentioned, the number of vertices and edges are respectively denoted by $n(G)$ and $m(G)$. The adjacency matrix $A = (a_{ij})$ of G is a $(0, 1)$ -square matrix of order n whose (i, j) -entry is equal to 1, if v_i is adjacent to v_j and equal to 0, otherwise. The spectrum of the adjacency matrix is called the adjacency spectrum of G . If $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ are the adjacency eigenvalues of G , the energy [13] of G is defined as $E(G) = \sum_{i=1}^n |\lambda_i|$. This quantity introduced by I. Gutman has noteworthy chemical applications and its mathematical aspect is well developed (see [17]).

Let $D(G) = \text{diag}(d_1, d_2, \dots, d_n)$ be the diagonal matrix associated to G , where $d_i = d(v_i) = \deg(v_i)$, for all $i = 1, 2, \dots, n$. The matrices $L(G) = D(G) - A(G)$ and $Q(G) = D(G) + A(G)$ are respectively, called the Laplacian and the signless Laplacian matrices and their spectrum are respectively, called the Laplacian spectrum and the signless Laplacian spectrum of the graph G . Both the matrices $L(G)$ and $Q(G)$ are real symmetric, positive semi-definite matrices, therefore their eigenvalues are non-negative real numbers. Let $0 = \mu_n \leq \mu_{n-1} \leq \dots \leq \mu_1$ and $0 \leq q_n \leq q_{n-1} \leq \dots \leq q_1$ be the Laplacian spectrum and signless Laplacian spectrum of G , respectively. It is well known that $\mu_n = 0$ with multiplicity equal to the number of connected components of G , and $\mu_{n-1} > 0$ if and only if G is connected. The positive inertia of a graph G is the number of positive adjacency eigenvalues of G and its negative inertia is the number of negative adjacency eigenvalues. The sum of positive and negative inertia of G is always equal to the rank of G .

The motivation for Laplacian energy comes from graph energy [13,17]. The Laplacian energy of a graph G as put forward by Gutman and Zhou (see [15]) is defined as $LE(G) = \sum_{i=1}^n \left| \mu_i - \frac{2m}{n} \right|$. This quantity, which is an extension of graph-energy concept, has found remarkable chemical applications beyond the molecular orbital theory of conjugated molecules (see [24]). For recent development on $LE(G)$ see [6,10,11,21,22] and the references therein. It is easy to see that

$$\text{tr}(L(G)) = \sum_{i=1}^n \mu_i = 2m = \sum_{i=1}^n q_i = \text{tr}(Q(G)),$$

where tr is the trace.

In analogy to Laplacian energy, the signless Laplacian energy $QE(G)$ of G is defined as $QE(G) = \sum_{i=1}^n \left| q_i - \frac{2m}{n} \right|$. If σ , $1 \leq \sigma \leq n-1$, are the number of signless Laplacian eigenvalues greater than or equal to average degree $\bar{d} = \frac{2m}{n}$, then by a simple observation, it follows that

$$QE(G) = \sum_{i=1}^n \left| q_i - \frac{2m}{n} \right| = 2S_{\sigma}^{+}(G) - \frac{4m\sigma}{n} = \max_{1 \leq i \leq n-1} \left\{ 2S_i^{+}(G) - \frac{4mi}{n} \right\}, \quad (1)$$

where $S_{\sigma}^{+}(G) = \sum_{i=1}^{\sigma} q_i$ is the sum of σ largest signless Laplacian eigenvalues of G .

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