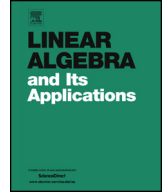




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# On singularities of third secant varieties of Veronese embeddings <sup>☆</sup>



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### ABSTRACT

In this paper we study singularities of third secant varieties of Veronese embedding  $v_d(\mathbb{P}^n)$ , which corresponds to the variety of symmetric tensors of border rank at most three in  $(\mathbb{C}^{n+1})^{\otimes d}$ .

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# 1. Introduction

For a projective algebraic variety  $X \subset \mathbb{P}W$ , the  $k$ -th secant variety  $\sigma_k(X)$  is defined by

$$\sigma_k(X) = \overline{\bigcup_{x_1 \cdots x_k \in X} \mathbb{P}\langle x_1 \cdots x_k \rangle} \subset \mathbb{P}W \quad (1.1)$$

where  $\langle x_1 \cdots x_k \rangle \subset W$  denotes the linear span of the points  $x_1 \cdots x_k$  and the overline denotes Zariski closure. Let  $V$  be an  $(n + 1)$ -dimensional complex vector space and  $W = S^d V$  be the subspace of symmetric  $d$ -way tensors in  $V^{\otimes d}$ . Equivalently, we can also think of  $W$  as the space of homogeneous polynomials of degree  $d$  in  $n + 1$  variables. When  $X$  is the Veronese embedding  $v_d(\mathbb{P}V)$  of rank one symmetric  $d$ -way tensors over  $V$  in  $\mathbb{P}W$ , then  $\sigma_k(X)$  is the variety of symmetric  $d$ -way tensors of border rank at most  $k$  (see Subsection 2.1 for terminology and details).

If  $X$  is an irreducible variety and  $\sigma_k(X)$  its  $k$ -secant variety, then it is well known that

$$\text{Sing}(\sigma_k(X)) \supseteq \sigma_{k-1}(X) \quad (1.2)$$

(e.g. see [3, Corollary 1.8]). Equality holds in many basic examples, like determinantal varieties defined by minors of a generic matrix, but the strict inequality also holds for some other tensors (e.g. just have a look at [12, Corollary 7.17] for the case  $\sigma_2(X)$  when  $X$  is the Segre embedding  $\mathbb{P}V_1 \times \cdots \times \mathbb{P}V_r$  or [2, Fig. 1, p. 18] for the third secant variety of Grassmannian  $\mathbb{G}(2, 6)$ ).

Therefore, it should be very interesting to compute more cases and to give a general treatment about singularities of secant varieties. Further, the knowledge of singular locus is known to be very crucial to the so-called *identifiability problem*, which is to determine uniqueness of a tensor decomposition (see [6, Theorem 4.5]). It has recently been paid more attention in this context. In this paper, we deal with the case of third secant variety of Veronese embeddings,  $\sigma_3(v_d(\mathbb{P}V))$ .

From now on, let  $X$  be the Veronese variety  $v_d(\mathbb{P}V)$  in  $\mathbb{P}S^d V = \mathbb{P}^N$  with  $N = \dim_{\mathbb{C}} S^d V - 1 = \binom{n+d}{n} - 1$ . One could ask the following problem:

**Problem 1.1.** Let  $V = \mathbb{C}^{n+1}$ . Determine for which triple  $(k, d, n)$  it does hold that

$$\text{Sing}(\sigma_k(v_d(\mathbb{P}V))) = \sigma_{k-1}(v_d(\mathbb{P}V))$$

for every  $k \geq 2, d \geq 2$  and  $n \geq 1$  or describe  $\text{Sing}(\sigma_k(v_d(\mathbb{P}V)))$  if it is not the case.

We'd like to remark here that our question is a set-theoretic one. First, it is classical that the answer to Problem 1.1 is true for the binary case (i.e.  $n = 1$ ) (see e.g.

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