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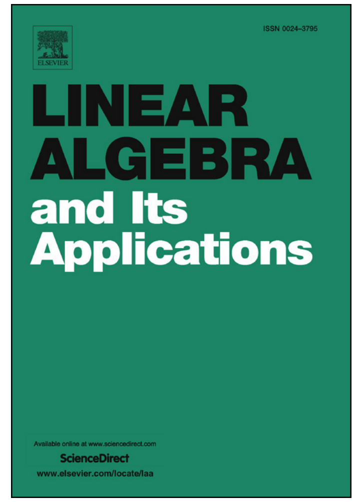

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ON THE BISYMMETRIC NONNEGATIVE INVERSE EIGENVALUE PROBLEM

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Abstract. The problem of finding conditions for a list of n complex numbers to be the eigenvalues of an $n \times n$ nonnegative matrix is called *the nonnegative inverse eigenvalue problem* (NIEP). The SNIEP (BNIEP) is the NIEP when the desired matrices are symmetric (bisymmetric). Recently, some sufficient conditions for the BNIEP are given by Julio and Soto in [6]. In this article, we study the BNIEP for $n = 4$ and prove that the BNIEP and SNIEP are different for $n = 6$. Then we give some sufficient conditions for the BNIEP and for the NIEP for normal centrosymmetric matrices related to the sufficient conditions of Julio and Soto and we also provide certain sufficient conditions for the BNIEP analogous to the sufficient conditions for the NIEP given by Borobia in [1]. Finally, we give sufficient conditions for the BNIEP when the prescribed diagonal entries are required.

1. Introduction

The problem of finding necessary and sufficient conditions for n complex numbers to be the spectra of a nonnegative matrix is called the *nonnegative inverse eigenvalue problem* (NIEP). This problem was first studied by Suleimanova in [15] and then several authors have extensively studied this problem. However, the problem remains open for $n \geq 5$. If we consider this problem only for the list of n real numbers, then the problem will be called the *real nonnegative inverse eigenvalue problem* (RNIEP). A set of n complex numbers is said to be *realizable* if there is an $n \times n$ nonnegative matrix having these complex numbers as its eigenvalues. First, we give three important results on certain sufficient conditions for the RNIEP.

Theorem 1.1 (Suleimanova [15], 1949). *If $\lambda_0 \geq 0 \geq \lambda_1 \geq \ldots \geq \lambda_n$ are real numbers such that $\sum_{i=0}^{n} \lambda_i \geq 0$, then $\{\lambda_0, \lambda_1, \ldots, \lambda_n\}$ is realizable.*

Theorem 1.2 (Kellogg [7], 1971). *Let $\lambda_0 \geq \lambda_1 \geq \ldots \geq \lambda_M \geq 0 > \lambda_{M+1} \geq \lambda_{M+2} \geq \ldots \geq \lambda_n$ be real numbers and let*

$$K = \{i \in \{1, 2, \ldots, \min\{M, n-M\}\} \mid \lambda_i + \lambda_{n-i+1} < 0\}.$$

If the following condition (Kellogg's condition) *holds:*

$$\lambda_0 + \sum_{i \in K, i<k} (\lambda_i + \lambda_{n-i+1}) + \lambda_{n-k+1} \geq 0 \qquad \text{for all } k \in K,$$

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