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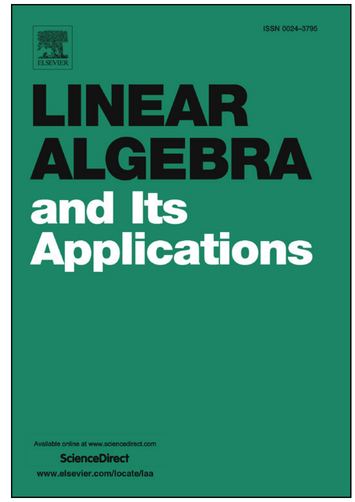

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Complementarity eigenvalue analysis of connected graphs

Alberto Seeger[1]

Abstract. This work concerns the spectral analysis of connected graphs from a non-traditional point of view. Instead of the usual eigenvalues of the adjacency matrix A_G of a graph G under consideration, we compute and analyze the complementarity eigenvalues of A_G. The complementarity eigenvalues of a general square matrix are defined in terms of a certain complementarity system relative to the componentwise ordering. The complementarity eigenvalues of A_G form the so-called complementarity spectrum of G. In general, the structure of a connected graph is better discriminated in terms of its complementarity spectrum than in terms of its usual spectrum. This observation is one of the leading motivation behind our work.

Mathematics Subject Classification: 05C50, 15A42.
Key words: complementarity eigenvalue analysis, connected graphs, spectral radius, second largest complementarity eigenvalue, connected induced subgraphs.

1 Introduction

All graphs considered are finite, undirected, simple, and unlabeled. A standard reconstruction problem of graph theory is to identify a graph G from its characteristic polynomial

$$\varphi(\lambda, G) = \det(\lambda I - A_G)\,. \tag{1}$$

Here, A_G is the adjacency matrix of G and I is the identity matrix of appropriate order. Recall that the adjacency matrix of a graph is unique up to a permutation similarity transformation. The roots or zeros of the monic polynomial (1) are called the eigenvalues of G. As it is well known, such roots provide useful information on the structure of G, but the graph itself may not be fully determined from its spectrum alone. Two or more graphs with the same characteristic polynomial are said to be cospectral. Pairs of cospectral graphs are abundant and easy to find, even if we restrict the attention to trees or other particular classes of connected graphs; cf. [7, 8, 9, 14, 20].

Since the eigenvalues of the adjacency matrix A_G may poorly represent the graph G itself, it is natural to introduce an alternative collection of parameters that discriminate better the nature of the graph under consideration. Changing the adjacency matrix by the Laplacian matrix L_G of the graph G does not fix the problem caused by cospectrality. Indeed, bringing the eigenvalues of L_G into the picture is still insufficient to determine G in a unique way. This work proposes to have a look not at the usual spectrum of A_G but at the complementarity spectrum of this matrix.

Definition 1. *Let A be a matrix of order n. A real λ is a complementarity eigenvalue of A if there exists a nonzero vector $x \in \mathbb{R}^n$ satisfying the complementarity system*

$$0 \leq x \perp (Ax - \lambda x) \geq 0, \tag{2}$$

where $\perp$ stands for orthogonality. The complementarity spectrum of a graph G, denoted by $\Pi(G)$, is the set of complementarity eigenvalues of the adjacency matrix of G.

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