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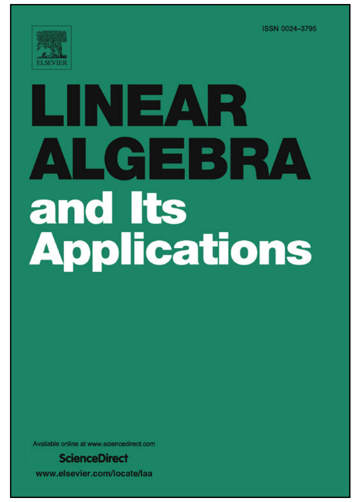

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WHEN IS EVERY LINEAR TRANSFORMATION A SUM OF AN IDEMPOTENT ONE AND A LOCALLY NILPOTENT ONE?

GAOHUA TANG, GUOLI XIA AND YIQIANG ZHOU

Abstract. For a semisimple module M over a ring R with $R/J(R)$ Boolean, every endomorphism of M is a sum of an idempotent endomorphism and a locally nilpotent endomorphism. As a consequence, it is proved that, for a vector space V over a division ring D, every linear transformation of V is a sum of an idempotent linear transformation and a locally nilpotent linear transformation if and only if $D \cong \mathbb{F}_2$. This can be seen as an answer to the "local" version of a question raised by Breaz et al. in [1] on nil-cleanness of the ring of linear transformations of an infinite dimensional vector space.

1. Introduction

An element of a ring is nil-clean if it is a sum of an idempotent and a nilpotent element, and a ring is nil-clean if every element is a nil-clean element (see Diesl [2]). In [1], Breaz et al. proved that for a field F, the matrix ring $\mathbb{M}_n(F)$ is nil-clean if and only if $F \cong \mathbb{F}_2$. Toward extending this result, the authors in [1] raised two questions: (i) Does the result hold for a division ring? (ii) Is the ring of linear transformations of a countably infinite dimensional vector space over $\mathbb{F}_2$ a nil-clean ring? While the first question was affirmatively answered in [3], the second one remains open.

Recall that an endomorphism f of a module M is locally nilpotent if for any $x \in M$, $f^n(x) = 0$ for some $n > 0$. It is easily seen that, for a finitely generated module M, an endomorphism of M is locally nilpotent if and only if it is nilpotent. Thus, the aforementioned results can be phrased as that, for a finite dimensional vector space V over a division ring D, every linear transformation of V is a sum of an idempotent linear transformation and a locally nilpotent linear transformation if and only if $D \cong \mathbb{F}_2$. Also, the question of Breaz et al. [1] has the following "local" version: For a vector space V with arbitrary dimension over

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