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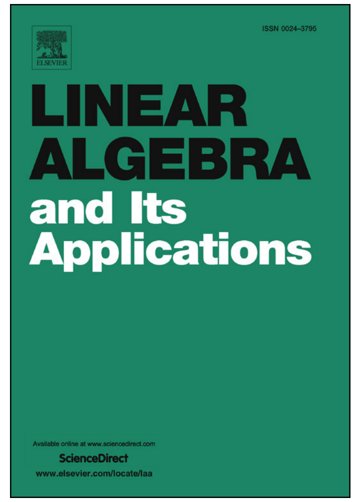

Implications of losing Hermiticity in quantum mechanics

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Implications of losing Hermiticity in quantum mechanics

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Abstract

In this note, we revisit a system of one self-interacting boson, described by a non-Hermitian Hamiltonian H acting on an infinite dimensional Hilbert space $\mathcal{H}$. We determine the eigenfunctions of the Hamiltonian and of its adjoint, which constitute complete biorthogonal sets. The probabilistic interpretation of quantum mechanics is not compatible with the metric inherited from $\mathcal{H}$, and attempts to overcome this problem are presented. Consequences of losing self-adjointness in the quantum mechanical context are discussed and the necessity of a careful mathematical analysis of unbounded operators is emphasized.

"In spite of the quasi-Hermiticity (without bounded inverse of Q), there is for instance no hope of building functional calculus that would follow more or less the same pattern of the self-adjoint operators."

Dieudonné

AMS subject classification: 34K08, 47–02, 46N50

Key words: Hamiltonian, unbounded operator, Complete biorthogonal sets, Riesz basis, Harmonic oscillator

1 Introduction

In quantum mechanics, the states of a particle, or system of particles, are represented by vectors in a separable Hilbert space $\mathcal{H}$, endowed with an inner product $\langle \cdot, \cdot \rangle$ and related norm $\| \cdot \|$. Measurable physical quantities are represented by self-adjoint, or Hermitian (in the terminology of von Neumann), operators in

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