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Block Kronecker ansatz spaces for matrix polynomials

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In this paper, we introduce a new family of equations for matrix pencils that may be utilized for the construction of strong linearizations for any square or rectangular matrix polynomial. We provide a comprehensive characterization of the resulting vector spaces and show that almost every matrix pencil therein is a strong linearization regardless whether the matrix polynomial under consideration is regular or singular. These novel "ansatz spaces" cover all block Kronecker pencils as introduced in [6] as a subset and therefore contain all Fiedler pencils modulo permutations. The important case of square matrix polynomials is examined in greater depth. We prove that the intersection of any number of block Kronecker ansatz spaces is never empty and construct large subspaces of block-symmetric matrix pencils among which still almost every pencil is a strong linearization. Moreover, we show that the original ansatz spaces $\mathbb{L}_1$ and $\mathbb{L}_2$ may essentially be recovered from block Kronecker ansatz spaces via pre- and postmultiplication, respectively, of certain constant matrices.

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1. Introduction

The linearization of (non)square matrix polynomials

$$P(\lambda) = \sum_{i=0}^{d} P_i \lambda^i, P_i \in \mathbb{R}^{m \times n}$$

has received much attention in the last ten years, motivated at least in part by the ground-breaking paper [12]. In that paper, three vector spaces $\mathbb{L}_1$, $\mathbb{L}_2$ and $\mathbb{DL}$ of potential linearizations (called "ansatz spaces") for square matrix polynomials $P(\lambda)(m = n)$ have been introduced. The spaces $\mathbb{L}_1$, $\mathbb{L}_2$ generalize the companion form of the first and second kind, resp.,

$$\mathbb{L}_1(P) = \{\mathcal{L}(\lambda) = \lambda X + Y \in \mathbb{R}[\lambda]^{nd \times nd} \mid \mathcal{L}(\lambda)\big(\Lambda_{d-1} \otimes I_n\big) = v \otimes P(\lambda), v \in \mathbb{R}^d\},$$

$\mathbb{L}_2(P) = \{\mathcal{L}(\lambda)^T \mid \mathcal{L}(\lambda) \in \mathbb{L}_1(P^T)\}$ while the double ansatz space

$$\mathbb{DL}(P) = \mathbb{L}_1(P) \cap \mathbb{L}_2(P) \tag{1}$$

is their intersection. Here Λ_j is the vector of the elements of the standard basis; $\Lambda_j := \Lambda_j(\lambda) = [\ \lambda^j\ \lambda^{j-1}\ \cdots\ \lambda\ 1\ ]^T \in \mathbb{R}[\lambda]^{j+1}$ for any integer $j \geqslant 0$. A thorough discussion of these spaces can be found in [12] and [10], see [6] for more references. In particular, it is discussed in [12] that almost all pencils in these spaces are linearizations of $P(\lambda)$ and in [10] that any matrix pencil in $\mathbb{DL}(P)$ is block-symmetric.

The second main source of linearizations are Fiedler pencils $F_\sigma(\lambda)$. Unlike the linearizations from the vector spaces discussed above, these can be defined not only for square, but also for rectangular matrices [5]. These pencils are defined in an implicit way, either in terms of products of matrices for square polynomials or as the output of a symbolic algorithm for rectangular matrices, see [6, Section 4] for a definition, a summary of their properties and references to further work.

In [6, Section 5] the family of block Kronecker pencils is introduced, which include all of the Fiedler pencils (modulo permutations). For an arbitrary matrix pencil $M_0 + \lambda M_1 \in \mathbb{R}[\lambda]^{(\eta+1)m \times (\epsilon+1)n}$ any matrix pencil of the form

$$\mathcal{N}(\lambda) = \left[\begin{array}{c|c} M_0 + \lambda M_1 & L_\eta^T \otimes I_m \\ \hline L_\epsilon \otimes I_n & 0_{\epsilon n \times \eta m} \end{array}\right] \in \mathbb{R}[\lambda]^{((\eta+1)m+\epsilon n) \times ((\epsilon+1)n+\eta m)} \tag{2}$$

is called an (ϵ, n, η, m)-block Kronecker pencil, or simply, a block Kronecker pencil. Here,

$$L_\kappa = L_\kappa(\lambda) := \begin{bmatrix} -1 & \lambda & & & \\ & -1 & \lambda & & \\ & & \ddots & \ddots & \\ & & & -1 & \lambda \end{bmatrix} \in \mathbb{R}[\lambda]^{\kappa \times (\kappa+1)}. \tag{3}$$