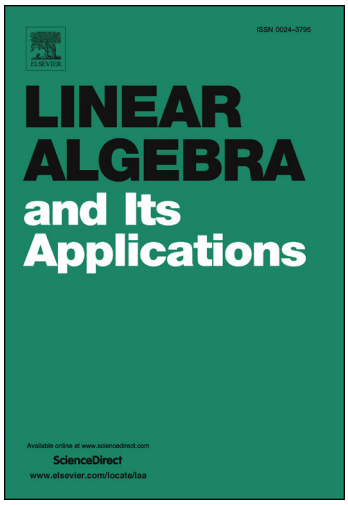

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Structured low rank decomposition of multivariate Hankel matrices

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Abstract

We present a new algorithm for the decomposition of a multivariate Hankel matrix H_σ as a sum of Hankel matrices of small rank. This decomposition corresponds to the decomposition of its symbol σ as a sum of polynomial-exponential series. By exploiting the properties of the associated Artinian Gorenstein quotient algebra $\mathcal{A}_\sigma$, we obtain new ways to compute the frequencies and the weights of the decomposition from generalized eigenvectors of sub-matrices of H_σ. The new method is a multivariate generalization of the so-called Pencil method for solving Prony-type problems. We analyse its numerical behaviour in the presence of noisy input moments. We describe rescaling techniques and Newton iterations, which improve the numerical quality of the reconstruction and show their impact for correcting errors on input moments.

AMS classification: 14Q20, 68W30, 47B35, 15B05
Keywords: Hankel; polynomial; exponential series; low rank decomposition; eigenvector; Singular Value Decomposition.

1. Introduction

Structured matrices such as Toeplitz or Hankel matrices appear in many problems. They are naturally associated to operations on polynomials or series [Fuh12]. The correlation with polynomial algebra can be exploited to accelerate matrix computations [BP94]. The associated algebraic model provides useful information on the problem to be solved or the phenomena to be analysed. Understanding its structure often yields a better insight on the problem and its solution. In many cases, an efficient way to analyze the structure of the underlying models is to decompose the structured matrix into a sum of low rank matrices of the same structure. This low rank decomposition has applications in many domains [Mar12] and appears under different formulations [Lan11, BCMT10, BBCM13].

In this paper, we study specifically the class of Hankel matrices. We investigate the problem of decomposing a Hankel matrix as a sum of indecomposable Hankel matrices of low rank. Natural questions arise. What are the indecomposable Hankel matrices? Are they necessarily of rank 1 ? How to compute a decomposition of a Hankel matrix as a sum of indecomposable Hankel matrices? Is the structured low rank decomposition of a Hankel matrix unique ?

These questions have simple answers for non-structured or dense matrices: The indecomposable dense matrices are the matrices of rank one, which are the tensor product of two vectors. The Singular Value Decomposition of a dense matrix yields a decomposition as a minimal sum of rank one matrices, but this decomposition is not unique.

It turns out that for the Hankel structure, the answers to these questions are not so direct and involve the analysis of the so-called symbol associated to the Hankel matrix. The symbol is a formal power series defined from the coefficients of the Hankel matrix. As we will see, the structured decomposition of an Hankel matrix is closely related to the decomposition of the symbol as a sum of polynomial-exponential series.

The decomposition of the symbol of a Hankel matrix is a problem, which has a long history. The first work on this problem is probably due to Gaspard-Clair-François-Marie Riche de Prony [Bar95]. He proposed a method to reconstruct a sum of exponentials from the values at equally spaced data points, by computing a polynomial in the kernel of a Hankel matrix, and deducing the decomposition from the roots of