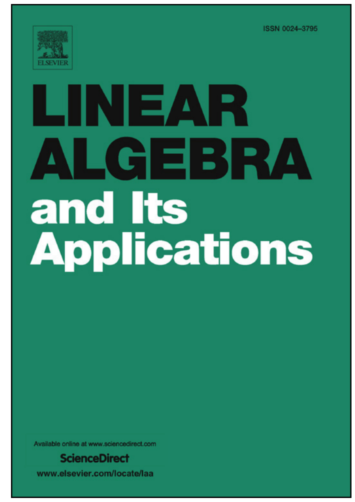

Decompositions of linear spaces induced by bilinear maps

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DECOMPOSITIONS OF LINEAR SPACES INDUCED BY BILINEAR MAPS

ANTONIO J. CALDERÓN MARTÍN

ABSTRACT. Let V be an arbitrary linear space and $f : V \times V \to V$ a bilinear map. We show that, for any choice of basis $\mathcal{B}$ of V, the bilinear map f induces on V a decomposition

$$V = \bigoplus_{j \in J} V_j$$

as a direct sum of linear subspaces, which is f-orthogonal in the sense

$$f(V_j, V_k) = 0$$

when $j \neq k$, and in such a way that any V_j is strongly f-invariant in the sense

$$f(V_j, V) + f(V, V_j) \subset V_j.$$

We also characterize the f-simplicity of any V_j. Finally, an application to the structure theory of arbitrary algebras is also provided.

Keywords: Linear space, bilinear map, orthogonality, invariant subspace, decomposition theorem.

2010MSC: 15A03, 15A21, 15A69, 15A86.

1. INTRODUCTION

We begin by noting that throughout the paper all of the linear spaces V considered are of arbitrary dimension and over an arbitrary base field $\mathbb{F}$.

The paper is organized as follows. In the second section we develop all of the techniques needed to get our main results. We begin by introducing connection techniques, previously used in different algebraic contexts (see [1, 2, 3, 4, 5, 6, 7]), in the framework of linear spaces V. As a consequence, we get that any choice of basis $\mathcal{B}$ of V gives rise to a first decomposition of V as an f-orthogonal direct sum of linear subspaces. In order to improve this decomposition we introduce an adequate equivalence relation on the above family of linear subspaces, which allows us to get our first main result asserting that V decomposes as an f-orthogonal direct sum of strongly f-invariant linear subspaces. In Section three it is discussed the relation among the previous decompositions of V given by different choices of bases of V. It is shown that if two basis $\mathcal{B}$ and $\mathcal{B}'$ of V belong to the same orbit under an action of a certain subgroup of $\mathrm{GL}(V)$ on the set of all of the basis of V, then they give rise to isomorphic decompositions of V. In Section four we prove that any of the linear subspaces in the decompositions of V given in Section two is f-simple if and only if its radical is zero and it admits an i-division basis. Finally, in Section five an application of the previous results to the the structure theory of arbitrary algebras is provided.

Let us recall some concepts related to a linear space V endowed with a bilinear map.

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