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Robustness and perturbations of minimal bases

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A B S T R A C T

Polynomial minimal bases of rational vector subspaces are a classical concept that plays an important role in control theory, linear systems theory, and coding theory. It is a common practice to arrange the vectors of any minimal basis as the rows of a polynomial matrix and to call such matrix simply a minimal basis. Very recently, minimal bases, as well as the closely related pairs of dual minimal bases, have been applied to a number of problems that include the solution of general inverse eigenstructure problems for polynomial matrices, the development of new classes of linearizations and ℓ-ifications of polynomial matrices, and backward error analyses of complete polynomial eigenstructure problems solved via a wide class of strong linearizations. These new applications have revealed that although the algebraic properties of minimal bases are rather well understood, their robustness and the behavior of the corresponding dual minimal bases under perturbations have not yet been explored in the literature, as far as we know. Therefore, the main purpose of this paper is to study in detail when a minimal basis $M(\lambda)$ is robust under perturbations, i.e., when all the polynomial matrices in a neighborhood of $M(\lambda)$ are minimal

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bases, and, in this case, how perturbations of $M(\lambda)$ change its dual minimal bases. In order to study such problems, a new characterization of whether or not a polynomial matrix is a minimal basis in terms of a finite number of rank conditions is introduced and, based on it, we prove that polynomial matrices are generically minimal bases with very specific properties. In addition, some applications of the results of this paper are discussed.

1. Introduction

Minimal bases, formed by vectors with polynomial entries, of a rational vector subspace were made popular in standard references of control theory and linear systems theory as those of Wolovich [23], Forney [13], and Kailath [17], although all three of them cite earlier work for some theoretical developments on the so-called *minimal polynomial bases*. For instance, one can read in [17, p. 460] the following sentence: *I.C. Gohberg pointed out to the author that the significance of minimal bases was perhaps first realized by J. Plemelj in 1908 and then substantially developed in 1943 by N.I. Mushkelishvili and N.P. Vekua.* This means that this paper deals with an almost 110 years old classical mathematical notion. However, the discovery of the importance of this concept in applications had to wait until the 1970s, when the contributions of authors such as Wolovich, Forney, Kailath, and others, provided computational schemes for constructing a minimal basis from an arbitrary polynomial basis, and showed the key role that this notion plays in multivariable linear systems. These systems could be modeled by rational matrices, polynomial matrices, or linearized state-space models, and had tremendous potential for solving analysis and design problems in control theory as well as in coding theory. The reader is referred to [13,17] and the references therein for more information on minimal bases and their applications, and also to the brief revision included in the Section 2 of this paper. Moreover, many papers have been published after [17] on the computation and applications of minimal bases and some of them can be found in the references included in [3].

Very recently, minimal bases, and the closely related notion of pairs of dual minimal bases, have been applied to the solution of some problems that have attracted the attention of many researchers in the last fifteen years. For instance, minimal bases have been used (1) in the solution of inverse complete eigenstructure problems for polynomial matrices (see [7,8] and the references therein), (2) in the development of new classes of linearizations and ℓ-ifications of polynomial matrices [5,9–12,18,22], which has allowed to recognize that many important linearizations commonly used in the literature are constructed via dual minimal bases (including the classical Frobenius companion forms), (3) in the explicit construction of linearizations of rational matrices [1], and (4) in the backward error analysis of complete polynomial eigenvalue problems solved via the so-

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